Income Inequality, Productivity, and International Trade

Wen-Tai Hsu Lin Lu Pierre M. Picard*†

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Abstract

This paper discusses the effect of income inequality on selection and aggregate productivity in a general equilibrium model with non-homothetic preferences. It shows the existence of a negative relationship between the number and quantity of products consumed by an income group and the earnings of other income groups. It also highlights the negative effect of mean-preserving spread of income on aggregate productivity through the softening of firms' selection. This effect is however mitigated in the presence of international trade. In a quantitative analysis, it is shown that a too large mean-preserving spread of income may harm the rich as it raises firms' markups on her purchases. This is contrary to the general belief that income inequality benefits the rich.

*Hsu: School of Economics, Singapore Management University. 90 Stamford Road, Singapore 178903. Email: wentaihsu@smu.edu.sg; Lu: Department of Economics, Tsinghua University. Email: lulin@sem.tsinghua.edu.cn. Picard: CREA, University of Luxembourg. Email: pierre.picard@uni.lu.

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1 Introduction

Income inequality reappears as a hot social and economic issue in many developed countries (Atkinson, Piketty and Saez 2011, Piketty 2013). The majority of the economics literature has focused on studying the causes for income inequality, and technological progress and trade liberalization have been presented as two major driving forces.[1] In this paper, we ask a different question – how does income inequality affect aggregate economic performance and welfare in the context of an open economy? In particular, does there exist an equity-efficiency trade-off in the sense that an increase in income inequality (i.e., a decrease in equity) increases efficiency as measured by aggregate productivity? Or, could this be the other way around?

These questions has largely been ignored in the trade literature because of the usual premise of homothetic preferences (e.g. Krugman 1981, Melitz 2003[2]) or absence of income effects in the consumption of traded goods (e.g. Melitz and Ottaviano 2008). As those premises make most aggregate economic variables invariant to income redistribution there is no point to discuss its effect there. In contrast, the assumption of non-homothetic preferences allows to shed light on the effect of income inequality on aggregate productivity and welfare in the frameworks of the recent trade literature with firm heterogeneity and endogenous product variety *à la* Melitz (2003) and Melitz and Ottaviano (2008).

We first motivate our theoretical investigation by examining the conditional correlations between a country's TFP and its income inequality. Using a country-year panel data during 1996-2012, and using the Gini coefficient and top 10% income share of two measures of income inequality, we find significant and negative correlations of aggregate TFP with the two inequality measures, controlling for country and/or year fixed effects. Moreover, as the two major explanations for the cross-country differences in economic performance are institutions and geography (or market access), we also control for these two factors, and find that the negative correlation remains robust. In other words, even conditional on institution, geography, and history (the state of development of a country right before 1996 is subsumed into the country fixed-effect), income inequality provides an additional explanatory power on aggregate TFP of a country.

We propose a theoretical analysis in which income inequality and trade affect aggre-

[1]For skill-biased technical change, see, for example, Berman, Bound, and Machin (1998) and Acemoglu (2002). On the effect of globalization, see, for example, Grossman and Rossi-Hansberg (2008), Costinot and Vogel (2010), Helpman, Itskhoki, and Redding (2010), Behrens, Pokrovsky and Zhelobodko (2014), Grossman, Helpman, and Kircher (2017), Grossman and Helpman (2018), and Kim and Vogel (2018).

[2]In fact, this conclusion applies for all models in the model class characterized by Arkolakis, Costinot, and Rodriguez-Clare (2012).

gate productivity. We study a general equilibrium model in which firms have heterogeneous productivity and individuals are endowed with different skills and same Stone-Geary non-homothetic preferences.[3] The presence of various skill groups results in income inequality and lead to demand patterns varying with individuals' incomes. We concentrate on an economy with two income groups (rich and poor) not only for the sake of analytical tractability but also because of the recent focus on top and bottom income groups. For tractability, we impose a Pareto productivity distribution in some parts of the analysis.

To clarify the basic properties of the model, we first analyze a closed economy where each firm enters and designs a differentiated variety and then decides to exit or produce its variety according to an idiosyncratic unit production cost. Under the assumed preferences, the consumption choice of an individual is unambiguously represented by the choke price of her inverse demand function. This corresponds to the maximum price at which she is willing to purchase a first unit of a variety. In contrast to Melitz and Ottaviano (2008) where there is no income effect due to quasi-linear preference, choke prices in our model differ across income groups. The choke prices of the rich and poor groups are then sufficient statistics of the demands for the whole set of varieties in the economy. Moreover, the price elasticity of individuals' demand also varies with choke price. In particular, ceteris paribus, the richer income group faces lower price elasticity of demand. For this reason, firms' pricing behavior hinges upon income groups and firms separate in two sets: the set of firms that have low unit production cost and target all consumers with low prices and the set of firms that have high unit production cost and target only the rich consumers with high prices. This is readily illustrated by the example of posters and art paintings: while both goods have the same decorative functionality, the latter is much more costly to make (especially in terms of per unit quality). At the equilibrium, only richer individuals are willing to purchase the two goods to decorate their houses. At the equilibrium, the price of each variety follows the movement of the rich and poor's choke prices.

Income inequality affects the average productivity across firms. It indeed alters the prices of varieties through its effect on the rich and poor's equilibrium choke prices. We show that an increase in the rich group's income raises this group's choke price, but there is a *cross effect* that such increase in the rich's income reduces the poor's choke price. The rich group is willing to consume a wider set of varieties and entices new firms producing more costly varieties to enter. At the same time prices augment and the poor reduce the basket and the quantity of her purchases. On average, firms uses more input to produce

[3] The same preference is also used by Murata (2009) and Simonovska (2015).

their goods, which decreases the average productivity. Similar effect emerges when the poor group becomes poorer because the cross effect implies that the rich's choke price becomes larger. As a result, a mean-preserving spread implies a lower average productivity because the rich's choke price unambiguously increases, and there are on average more costly firms in the economy.

We secondly study the effect of trade liberalization in an open economy. We find that the negative effect of a mean-preserving spread on aggregate productivity is mitigated by trade liberalization. The intuition is that lower trade costs expand variety and induce tougher selection. Smaller trade costs lead to a tougher selection of firms in favor of those with lower production costs. Compared with autarky, the number of unsold varieties is larger within the global economy. Hence, when the rich gets a higher income, she spreads her consumption towards the wider set of unsold goods in the whole world rather than concentrates her purchases on the narrower set of domestic unsold goods. In the end, consumed goods are produced with lower costs. In other words, it is the productivity gains of globalization that mitigate the negative impact of income inequality on average productivity.

We conduct a quantitative analysis to further examine the properties that are difficult to obtain analytically. The above-mentioned analysis regarding average productivity is based on the unweighted average across firms. We examine how aggregate productivity (i.e., average productivity weighted by cost) reacts to mean-preserving spreads. In particular, when the poor become poorer, their consumption basket is more toward the varieties that are cheaper to produce. Can this force alter the previous result? The answer is no: we still find unambiguous decreases in aggregate productivity with mean-preserving spreads.

In the quantitative analysis, we set the rich group to be the top 10% income earners. In 2015, the income ratio between the two groups in the US is 7.9. Using equivalent variation as a "real" measure of utility change, we find that an income reallocation from the income ratio of 7.9 to 1 is equivalent to a 69% rise of the poor's *real income* and a 30% fall of the rich's. However, this result suggests that for a given amount of additional income, the improvement in welfare in real terms would be larger if such additional income is given to the poor than to the rich. Similarly, even assuming Benthamite social welfare function, in which case the social planner does not actually value equality in utility, our result shows that income reallocation from the rich to the poor is welfare improving.

Surprisingly, we also find that whereas mean-preserving spreads increase the rich's income, the effect on the rich's utility can actually fall when the income inequality is large. The reason behind this result is two-folds: increasing income inequality reduces aggregate productivity and increases markups when the rich/poor gain/lose presence in the market. On the gains from trade, we find that moving from autarky to a benchmark trade cost ($\tau = 1.7$) is equivalent to increases of the poor's and rich's real incomes by 8.9% and 3.6%, respectively. Trade liberalization therefore benefits more to the poor because the poor consume more heavily on traded goods.[4]

Our paper is closely related to the broad literature of heterogeneous firms and productivity that is pioneered by Melitz (2003) and Eaton and Kortum (2002). To our knowledge, our analysis is the first to offer new testable predictions about how income inequality affects firm selection and average productivity. In contrast to the traditional view of equity-efficiency tradeoff, Aghion et al. (1999) have highlighted reducing income inequality may promote economic growth through saving, investment and incentives. Matsuyama (2002) has studied the dynamic effect of income inequality on productivity in the context of homogeneous firms and learning by doing. Higher income inequality is detrimental to growth because it reduces the "mass of consumption" and therefore the dynamic productivity gains from learning by doing.[5] Through a different mechanism, our model shows that average productivity falls with inequality as it shuffles the mass of consumption from low-cost to high-cost goods.

This paper relates to the literature on the relationship between income heterogeneity and trade. Matsuyama (2000) and Fajgelbaum, Grossman and Helpman (2011) focus on the effect of income heterogeneity on the patterns of trade in contexts in which goods differ in some vertical attributes (quality or priority of consumption). Behrens and Murata (2012), Fajgelbaum and Khandelwal (2016), and McCalman (2018) make contributions on the welfare implications of trade liberalization for different income groups. Nevertheless, none of these studies discuss the effects of income heterogeneity on selection and productivity and how trade matters for these effects.

This paper is also related to the broad literature on the effect of nonhomothetic preference. It can be used to study pro-competitive effect and pricing to markets, such as in Simonovska (2015), Bertoletti, Etro, and Simonovska (2018), and Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018), on optimality in monopolistic competition models, such as Parenti, Ushchev, and Thisse (2017) and Dhingra and Morrow (2019), on structural change, such as in Comin, Lashkari, and Mestieri (2018), or on trade flows and patterns of trade, such as in Fieler (2011) and Matsuyama (2015). Even though there are income effects in these models, there is no differentiation of income within a country.

[4]A similar point was made by Fajgelbaum and Khandelwal (2016)

[5]Though not a main focus of their paper, Bertoletti and Etro (2016) show that income inequality decreases entry of firms.

The remainder of the paper is organized as follows. Section 2 provides an empirical motivation for our theoretical investigation. Section 3 lays out the model in the closed economy, and provides various comparative statics, with a focus on the effect of income. Section 4 extends the model to the open economy, and carries out similar analysis with a focus on the effect of trade liberalization. Section 5 provides a quantitative analysis of the effects on aggregate productivity and welfare. Section 6 concludes.

2 Empirical Motivation

To motivate our theory, this section provides suggestive evidence on the relationship between income inequality and productivity. This section presents essentially conditional correlations without attempting to establish a causal relation. As we are concerned with how productivity is related to income inequality, we control for two major factors affecting the level of development or technology of a country – institution and geography (i.e., market access).[6] Mainly, we ask the following question: conditional on institution and market access, is there a positive or negative correlation between income inequality and country-level productivity? We first describe our country-year panel data and empirical specification, and then presents the results.

Country-level productivity is measured by the total factor productivity (TFP) obtained from the Penn World Table (PWT) 9.0.[7] A special feature of the PWT data is that there are one measure of TFP for cross-country comparison (CTFP), where the TFP level of the USA is set to 1 for all years, and another by-country time-series measure (RTFPNA), where the TFP level is calculated relative to the country's 2011 level (hence TFP of each country at 2011 is set to 1). To utilize the panel data nature of our regressors, we construct a panel of TFPs in the following way. We calculate a country c's TFP at year t relative to the US' level at 2011:

$$\mathrm{TFP}_{c,t} \equiv \mathrm{CTFP}_{c,t} \times \mathrm{RTFPNA}_{\mathrm{USA},t}.$$

A concern of such a panel of TFPs is that if year fixed-effects are controlled, then the panel is essentially reduced to a pool of cross-section TFPs because $\mathrm{RTFPNA}_{\mathrm{USA},t}$ is the same for all countries for each given year; thus, in this case, we basically rely on the cross-sectional variations of the regressors to explain the variation in the TFP. We include specifications

[6] There is a vast literature regarding these two factors. For the role of institution, see, for examples, Acemoglu, Johnson and Robinson (2005), Levchenko (2007), and Acemoglu and Robinson (2012). For the role of geography and market access, see, for examples, Krugman (1991), Diamond (1997), Redding and Venables (2004), Redding and Sturm (2008).

[7] For the detailed account for Penn World Table 8.0 and 9.0, see Feesntra, Inklaar, Timmer (2015).

where (1) only country fixed-effects are controlled and (2) both country and year fixed-effects are controlled.

We use two measures for income inequality for the period of 1996-2012: the *Gini Coefficient* and the share of total income by the top 10 percent (*Top 10% Income Share*), both obtained from World Development Indicator.[8] Following the literature on institution, we use the rule of law as the measure for institutional quality of a country. The *Rule of Law* index is obtained from the Worldwide Governance Indicators by the World Bank.[9] Following the literature on economic geography, we define *Market Access* as trade–cost-discounted and price-deflated sum of market sizes around the world. We use the real market potential from CEPII's Market Potential database.[10]

We first peek at the simple correlation by plotting averages of log of TFP and averages of income inequality measures (the averages are taken over years). Panels (A) and (B) of Figure 1 plot the average *Gini Coefficient* and average *Top 10% Income Share*, respectively. There is a clear negative correlation between income inequality and TFP.

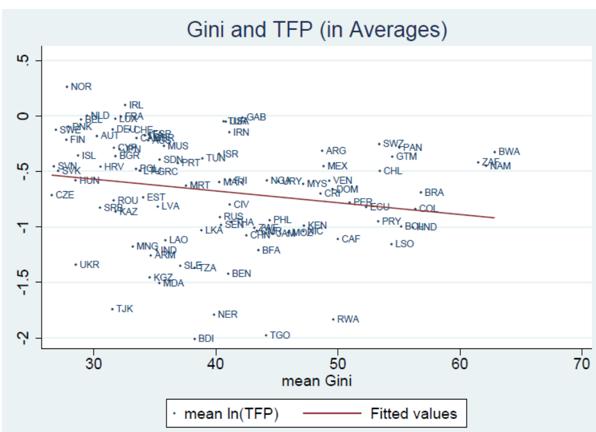

Figure 1(A)

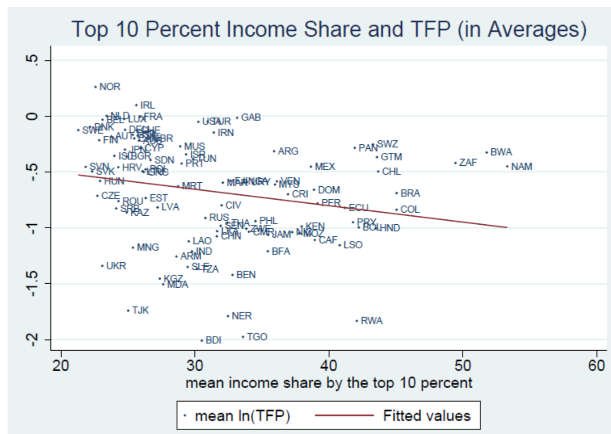

Figure 1(B)

We will estimate the following equation:

$$\ln \mathrm{TFP}_{it} = \beta_0 + \beta_1 \mathrm{Inequality}_{it} + \beta_2 X_{it} + d_i + d_t + \varepsilon_{it}$$

[8]We interpolate (but not extrapolate) the missing values based on available years for each country. The number of countries in the overlap between the income-inequality measure and the TFP varies year by year, but the number of countries is significantly smaller than 66 before 1996 and after 2012. Hence, we restrict the sample to 1996-2012 to have a more inclusive set of countries.

[9]This index is calculated by including several indicators which measure the extent to which agents have confidence in and abide by the rules of society, including perceptions of the incidence of crime, the effectiveness and predictability of the judiciary, and the enforceability of contracts. During 1996-2012, the *Rule of Law* is missing in 1997, 1999, and 2001, and hence we also interpolate for the missing values for these years.

[10]This is computed using Head and Mayer's (2004) method, which adjusts for the impacts of national borders on trade flows.

where Inequality$_{it}$ is the measure of inequality for country i in year t; X_{it} denotes our set of covariates (*Rule of Law* and log of *Market Access*); d_i and d_t are country and year fixed effects. Note that country fixed-effect includes history, i.e., the state of development of a country right before 1996. To account for potential serial correlations and heteroscedasticity, standard errors are clustered at country level.

The regression results are reported in Table 1. Columns (1) - (6) show results based on the Gini coefficient, whereas Columns (7) - (12) use the top 10% income shares. For each income inequality measure, the first column estimates the case where both year and country fixed-effects are controlled, whereas the second one estimates the case where the year fixed-effects are dropped for the reason explained above. The third and fourth columns are similar to the first two, except that now the *Rule of Law* is included as a control variable. The sample includes 1297 observations with 100 countries.[11] In the fifth and sixth columns, we further include the *Market Access* as a control. One caveat is that the sample size is reduced by half when the *Market Access* is included because the available years for this measure are only up to 2003.

[Insert Table 1 here.]

Both measures of inequality exhibit significant and negative correlation with TFP across most columns, consistent with the observation from Figure 1. When we control for institution and geography, the coefficients on income inequality in Columns (5-6) and (11-12) are significant at 10% level, whereas those in the other columns are significant at 1% level. The difference in significance levels is mostly due to the smaller sample sizes in Columns (5-6) and (11-12) due to data constraint. Also, the TFP of a country is higher when the country has larger effective market size and better institution, confirming the rationales of including these controls.

Columns (6) and (12) are our most preferred specification, as it allows both the time-varying and cross-sectional variations of the regressors to explain the variation in TFP. Taking Column (12) as a benchmark, we can interpret the coefficient of -0.807 as the following: if the *Top* 10% *Income Share* increases by 10 percentage points, the associated decline in TFP is about 7.7%, conditional on the same rule of law, market access, and year fixed-effects and country-specific time-invariant factors.

We next turn to our theory of how average productivity is affected by income inequality. We note here that our explanation is based on productivity selection, a mechanism that is distinct from institution or geography.

[11] Nevertheless, due to data constraint, it is an unbalanced panel.

3 Closed Economy

We present a model where a mass N of individuals are endowed with Stone-Geary preferences over a set of differentiated varieties $\omega \in \Omega$. Each individual h belongs to either the high or low income group $h \in \{L, H\}$ with income $s_h > 0$ and probability $\alpha_h \in (0, 1)$, $h \in \{L, H\}$ ($s_H > s_L$ and $\alpha_H + \alpha_L = 1$). Each firm produces a distinct variety ω. Firms differ in marginal cost $c(\omega)$ and face monopolistic competition.

3.1 Demand

An individual in the income group h chooses the consumption profile $q\left(.\right)$ that maximizes her utility $\int_{\omega\in\Omega} \ln\left(1 + q\left(\omega\right)/\bar{q}\right) \mathrm{d}\omega$[12] subject to her budget constraint $\int_{\omega\in\Omega} p\left(\omega\right) q\left(\omega\right) \mathrm{d}\omega = s_h$, where $\bar{q} > 0$ is a constant and the price profile $p\left(\cdot\right)$ is taken as given.[13] Without loss of generality we can normalize the unit of goods such that $\bar{q} = 1$. Her demand is equal to

$$q_h\left(\omega\right) = \frac{\hat{p}_h}{p\left(\omega\right)} - 1, \tag{1}$$

where

$$\hat{p}_h = \frac{s_h + P_h}{|\Omega_h|}, \tag{2}$$

and Ω_h is the set of goods that she consumes, $|\Omega_h| \equiv \int_{\omega\in\Omega_h} \mathrm{d}\omega$ is the measure of this set and

$$P_h \equiv \int_{\omega\in\Omega_h} p\left(\omega\right) \mathrm{d}\omega \tag{3}$$

is her (personal) price index over her consumptions (see Appendix A). The choke price $\hat{p}_h$ is the intercept of the individual demand curve; that is, the willingness to pay for the first unit of a good. At given exogenous set of prices and consumed goods, the choke price increases with larger income and aggregate price index and with smaller set of consumed goods. Yet, for any endogenous set of consumed goods, it is readily shown that the choke price is larger for higher income individuals: $\hat{p}_H > \hat{p}_L$.

[12] This is an affine transformation of the original Stone-Geary utility function $\int_{\omega\in\Omega} \ln\left(q\left(\omega\right) + \bar{q}\right) \mathrm{d}\omega$.

[13] We use an additive utility function that yields the Stone-Geary demand functions. Those are linear in income but do not exhibit expenditure proportionality (Pollak 1971). The linearity property is essential for the demand aggregation process below. The utility function belongs to the class of hierarichal preferences whereby the rich's basket of goods includes the poor's one, which has recieved good empirical support (Jackson 1984). Simonovska (2015) exploits this set-up to study international pricing-to-market under the assumption of homogenous income within a country.

The aggregate demand for each good ω with price $p(\omega) = p$ is given by

$$Q(p) \equiv \begin{cases} \alpha_H N \left(\frac{\hat{p}_H}{p} - 1 \right) & \text{if} \quad p \in [\hat{p}_L, \hat{p}_H) \\ N \left(\frac{\hat{p}_{HL}}{p} - 1 \right) & \text{if} \quad p \in [0, \hat{p}_L) \end{cases}, \tag{4}$$

where $\hat{p}_{HL} \equiv \alpha_H \hat{p}_H + \alpha_L \hat{p}_L$ is the average of individual choke prices ($\hat{p}_H \geq \hat{p}_{HL} \geq \hat{p}_L$). For a given price p, the aggregate demand $Q(p)$ is the same for all varieties because of the symmetric preferences. Because of the presence of two income groups, it has a convex kink at $p = \hat{p}_L$. The model mixes the properties of Mussa and Rosen's (1978) unit-purchase model with two income groups of consumers who demand one unit of an indivisible good with continuous-purchase models where goods are infinitely divisible.

The price elasticity is

$$\varepsilon(p) = -\frac{d \ln Q(p)}{d \ln p} = \begin{cases} \frac{\hat{p}_H}{\hat{p}_H - p} & \text{if} \quad p \in [\hat{p}_L, \hat{p}_H) \\ \frac{\hat{p}_{HL}}{\hat{p}_{HL} - p} & \text{if} \quad p \in [0, \hat{p}_L) \end{cases}.$$

Because $\hat{p}_H \geq \hat{p}_{HL}$, for a same price p, the elasticity is lower in the rich consumer segment.

3.2 Production

Labor is the only input. We consider two groups of individuals who differ only in the number of efficiency units they offer: a high (low) income individual is endowed with s_H (s_L) efficiency units of labor. In other words, we can interpret this as a difference in human capital. We choose labor efficiency unit as the numéraire so that s_H and s_L also measure high and low incomes.

Each firm produces and sells a unique variety ω under monopolistic competition. We assume the existence of a large pool of potential risk neutral entrants. By hiring f units of labor, each entrant obtains a distinct variety ω and gets a feasible production defined by an idiosyncratic marginal input in labor efficiency units, $c \in \mathbb{R}^+$. Given the above choice of numéraire, this also denotes the firm's marginal cost. The parameter c is drawn from a cumulative probability distribution $G : \mathbb{R}^+ \to [0, 1]$. We denote the mass of entrants by M. Therefore, each measure of goods $\mathrm{d}\omega$ is identical to the measure $M\mathrm{d}G(c)$.

Each firm maximizes its profit $\pi(c) = (p - c) Q(p)$ taking the choke prices $\hat{p}_L$ and $\hat{p}_H$ as given. Because the demand $Q(p)$ includes two segments, a firm can choose between targeting only the high income group or both income groups. Firms' optimal markup is readily given by

$$m(p) \equiv \frac{p(c)}{c} = \frac{\varepsilon(p)}{\varepsilon(p)-1} = \begin{cases} \frac{\hat{p}_H}{p} & \text{if} \quad p \in [\hat{p}_L, \hat{p}_H) \\ \frac{\hat{p}_{HL}}{p} & \text{if} \quad p \in [0, \hat{p}_L) \end{cases} .$$

Because $\hat{p}_{HL} \leq \hat{p}_H$, for a same price, markup is higher in the rich's market segment. The optimal price is given by

$$p^*(c) = \begin{cases} (\hat{p}_{HL} c)^{1/2} & \text{if} \quad c \leq \hat{c} \\ (\hat{p}_H c)^{1/2} & \text{if} \quad c > \hat{c} \end{cases} \tag{5}$$

and

$$\hat{c}^{1/2} \equiv \frac{(\hat{p}_{HL})^{1/2} - (\alpha_H \hat{p}_H)^{1/2}}{1 - \alpha_H^{1/2}}. \tag{6}$$

(see Appendix B). Except at $c = \hat{c}$, the optimal price is strictly concave increasing function of c. The concavity reflects that the presence of a pro-competitive effect whereby markups fall with higher cost c. Observe that because $\hat{p}_H > \hat{p}_{HL}$, the price jumps upward for the firm with cost c just above $\hat{c}$, reflecting a switch towards targeting the high income consumers. Note that, in a partial equilibrium where we change one choke price and take the other as fixed, we have

$$\frac{\partial \hat{c}}{\partial \hat{p}_H} < 0 \quad \text{and} \quad \frac{\partial \hat{c}}{\partial \hat{p}_L} > 0. \tag{7}$$

This means that the cutoff $\hat{c}$ falls when the rich gets higher income and her choke price rises. This is because their willingness to pay improves and more firms find it profitable to target them. By contrast, the cutoff rises when the poor become richer and their choke price rises. Targeting the entire population becomes more profitable.

In the product market equilibrium, each income group purchases the goods that are targeted to them. In particular, the low income consumers buy only the goods produced at cost in the range $[0, \hat{c}]$. This means that their choke price $\hat{p}_L$ satisfy $p^*(\hat{c} - 0) < \hat{p}_L < p^*(\hat{c} + 0)$. Given that all those goods are actually supplied by firms it must also be they have low enough cost, i.e. $\hat{c} < \hat{p}_L$. High income individuals purchase goods produced at costs in a range $[0, \hat{c}_H]$ with $\hat{c}_H > \hat{c}$. At the equilibrium, the price of the last purchased good is lower than their choke price: $p^*(\hat{c}_H) \leq \hat{p}_H$. Furthermore this good is supplied by a firm that makes zero profit. This implies that $\hat{p}_H = p^*(\hat{c}_H) = \hat{c}_H$. In Appendix B, we show that those conditions always hold. We summarize those points in the following proposition:

Proposition 1. *The equilibrium price $p^*(c)$ increases in marginal cost c and jumps up in c at $\hat{c}$. Low income consumers purchase goods produced at cost $c \in [0, \hat{c}]$ and high income consumers those at cost $c \in [0, \hat{p}_H]$, where $\hat{p}_H > \hat{c}$.*

It is worthwhile pointing out that the quantity $q(\omega)$ in the utility function can actually be interpreted as quality units if unit production cost c is also in terms of the cost per unit of quality. Namely, the fact that the rich purchase more costly goods in this model can be understood as they purchase goods with higher costs per unit of quality. Thus, our model is entirely consistent with the notion that the rich purchase high quality goods. In our model, the rich's willingness to pay for quality is, indeed, larger than poor's.

3.3 Equilibrium

Given Proposition 1, at the product market equilibrium, the choke prices can be written as

$$\hat{p}_L = \frac{s_L + P_L}{MG(\hat{c})}, \quad \hat{p}_H = \frac{s_H + P_H}{MG(\hat{p}_H)} \quad \text{and} \quad \hat{p}_{HL} = \alpha_H \hat{p}_H + \alpha_L \hat{p}_L$$

and the price indices as

$$P_L = (\hat{p}_{HL})^{1/2} \int_0^{\hat{c}} c^{1/2} M \mathrm{d}G(c) \quad \text{and} \quad P_H = P_L + \hat{p}_H^{1/2} \int_{\hat{c}}^{\hat{p}_H} c^{1/2} M \mathrm{d}G(c).$$

Eliminating price indices, these equilibrium conditions can be expressed as

$$e_H(\hat{p}_H, \hat{p}_L) - \frac{s_H}{M} = 0, \tag{8}$$

$$e_L(\hat{p}_H, \hat{p}_L) - \frac{s_L}{M} = 0, \tag{9}$$

where

$$e_H(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c),$$

$$e_L(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$

are the consumers' average expenditures per available variety while $\hat{c}$ is given by its definition (6). After some algebraic manipulations, one can simplify the equilibrium conditions (8) and (9) as

$$M = \frac{s_H}{e_H(\hat{p}_H, \hat{p}_L)} = \frac{s_L}{e_L(\hat{p}_H, \hat{p}_L)}. \tag{10}$$

Thus, consumers' expenditures per unit of income are equal across income groups and equal to the equilibrium mass of entrants . The product market equilibrium is defined by the solution of those two equations for the choke prices $(\hat{p}_H, \hat{p}_L)$. For a given M, the equilibrium choke prices are sufficient statistics of product market equilibrium consumption and production choices.

In the long run firms enter the market. Before entry, each entrant expects to cover her entry cost so that

$$\int_0^\infty \max\{\pi(c), 0\}\mathrm{d}G(c) = f,$$

where the profit $\pi(c)$ is given by $N\left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2$ if $c \leq \hat{c}$ and by $\alpha_H N\left(\hat{p}_H^{1/2} - c^{1/2}\right)^2$ if $c > \hat{c}$. Then, the entry condition writes as

$$\pi(\hat{p}_H, \hat{p}_L) = \frac{f}{N}, \tag{11}$$

where

$$\pi(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{p}_H} \max\left\{\left((\alpha_H\hat{p}_H + \alpha_L\hat{p}_L)^{1/2} - c^{1/2}\right)^2, \alpha_H\left(\hat{p}_H^{1/2} - c^{1/2}\right)^2\right\}\mathrm{d}G(c) \tag{12}$$

is the expected operational profit after entry. The general equilibrium is defined by the variables $\hat{p}_H$, $\hat{p}_L$ and M solving the equations in (8), (9) and (11). Finally, let the marginal cost distribution have a bounded support and finite mean:

$$G : [0, c_M] \to [0,1] \text{ such that } \mathrm{E}(c) = \int_0^{c_M} c\mathrm{d}G(c) < \infty. \tag{A0}$$

We prove the existence of a fixed point to the system of equations (8), (9) and (11):

Proposition 2. *There exists an equilibrium under (A0).*

Proof. See Appendix C. ■

A condition for the uniqueness of the general equilibrium can be found as follows. First note that the expected operational profit $\pi(\hat{p}_H, \hat{p}_L)$ is an increasing function of both choke prices. So, the entry condition describes a decreasing relationship between the two choke prices. Second, it can be seen that the second equality in (10) describes an increasing relationship between the two choke prices if the conditions $\partial e_h/\partial\hat{p}_h > 0$ and $\partial e_h/\partial\hat{p}_l < 0$ hold for any $h \neq l \in \{H, L\}$. Under those conditions, it is clear that the two relationships cross in a single point $(\hat{p}_H, \hat{p}_L)$ that yields the unique equilibrium. The main question is to verify that those conditions are true.

Using (6), it is easy to verify that the poor's expenditure increases with own choke price and falls with the rich's choke price: $\partial e_L/\partial \hat{p}_L > 0$ and $\partial e_L/\partial \hat{p}_H < 0$. The symmetric condition holds for the rich provided that firms do not change consumer segment targets. That is, if the cut-off cost $\hat{c}$ is fixed. However, by (6), the cut-off cost $\hat{c}$ falls ($\mathrm{d}\hat{c} < 0$) when $\hat{p}_H$ rises or $\hat{p}_L$ falls. Then a mass $-g(\hat{c})\mathrm{d}\hat{c} > 0$ of firms shift to the high income segment target, which reduces the rich's expenditure by the amount $\left(\hat{p}_H^{1/2} - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2}\right) \hat{c}^{1/2}$ $(-g(\hat{c})\mathrm{d}\hat{c})$. The change in firms' segment target therefore decreases the rich's expenditure and goes in the opposite direction of the effect of choke prices when $\hat{c}$ is fixed. Since this countervailing effect is proportional to the density $g(\hat{c})$, some smoothness property are required to guarantee that G is not misbehaved about $c = \hat{c}$. Let

$$\partial e_H/\partial \hat{p}_H > 0 \quad \text{and} \quad \partial e_H/\partial \hat{p}_L < 0. \tag{A1}$$

We then have the following:

Proposition 3. *The equilibrium exists and is unique if the cost distribution G satisfies (A0) and (A1).*

3.4 Income Distribution

We are interested in understanding how demands and choke prices are affected by changes in income levels of the two groups. Intuitively, an increase in the income of one group raises its willingness to pay, choke price and product demands. Since demand elasticity falls with higher income, markups and prices increase. Facing higher prices, the other group is enticed to diminish its consumption, which should be reflected by lower choke prices. In appendix D, we prove the following Lemma:

Lemma 1. *Under (A0) and (A1), a rise in the rich (resp. poor) group's skill and income raises its choke price and demands whereas it reduces the poor's (resp. rich's). Formally,*

$$\frac{\mathrm{d}\ln \hat{p}_h}{\mathrm{d}\ln s_h} = -\frac{\mathrm{d}\ln \hat{p}_h}{\mathrm{d}\ln s_l} > 0, \quad h \in \{H, L\}, \ell \neq h. \tag{13}$$

This has implications about the effect of income distribution on the average productivity and set of consumption goods. First, in this model, the two group incomes can be written as $s_H = \overline{s} + \alpha_L v$ and $s_L = \overline{s} - \alpha_H v$ where $\overline{s} \equiv \alpha_L s_L + \alpha_H s_H$ is the average income and $v \equiv s_H - s_L$ the income differential. Using this definition, a mean preserving spread of the income distribution is equivalent to a rise in v, holding $\overline{s}$, α_H and α_L constant. As a result, by (13), a mean preserving spread increases the choke price of the high income group. It indeed increases the high income and decreases the low income so that

$$\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}v}=\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H}\frac{\mathrm{d}\ln s_H}{\mathrm{d}v}+\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_L}\frac{\mathrm{d}\ln s_L}{\mathrm{d}v}=\frac{\overline{s}}{s_H s_L}\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H}>0. \tag{14}$$

Second, the average productivity is negatively related to the (unweighted) average cost in the economy, which is given by $\int_0^{\hat{p}_H} c\mathrm{d}G(c) / \int_0^{\hat{p}_H} \mathrm{d}G(c)$. The average cost moves in the same direction as the choke price $\hat{p}_H$ while the average productivity goes in the opposite direction. Hence, by (13), the average productivity falls with higher s_H. By (14), it also falls with a mean preserving spread of the income distribution (higher v). The point is that when the high income group gets richer, it consumes more varieties with high production cost, which raises the average cost and reduces the average productivity in the economy.

Finally, we investigate how the baskets of goods is altered after income changes. The basket of the poor's individual is given by the cut-off cost $\hat{c}$. This cost falls with a higher income for the rich and decreases with a higher income for the poor. We indeed have

$$\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_H}=\frac{\partial\ln\hat{c}}{\partial\ln\hat{p}_H}\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H}+\frac{\partial\ln\hat{c}}{\partial\ln\hat{p}_L}\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_H}<0,$$

where the inequality stems from (7) and (13). The increase in the rich's income raises her demand so that more firms target her and raise their prices. Higher prices then decreases the poor's demand and raises further the incentives to target the rich. It is readily verified that the opposite effect holds with a change in the poor's income: $\mathrm{d}\ln\hat{c}/\mathrm{d}\ln s_L>0$. As a consequence, a mean preserving spread of the income distribution reduces the cut-off cost $\hat{c}$. Indeed one readily checks that

$$\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}v}=\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_H}\frac{\mathrm{d}\ln s_H}{\mathrm{d}v}+\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_L}\frac{\mathrm{d}\ln s_L}{\mathrm{d}v}<0.$$

The mean preserving spread therefore reduces the relative measure of varieties consumed by the poor to the rich: that is, it reduces the ratio

$$\frac{MG(\hat{c})}{MG(\hat{p}_H)}=\frac{G(\hat{c})}{G(\hat{p}_H)}.$$

Proposition 4. *Suppose that the cost distribution G satisfies (A0) and (A1). Then, a mean-preserving spread of the income distribution (i) increases the choke price of the high income group, (ii) reduces the (unweighted) average productivity in the economy and (iii) reduces the set of goods consumed by the poor relative to that by the rich.*

Income redistribution policies have the opposite effect of mean-preserving spreads: they lower the choke price of the high income group and raises average productivity. This model yields a clear-cut answer as to how mean-preserving spread of income distributions affect aggregate economic performances. Such a result does not show up in a model under homothetic preference or under a quasi-linear preference (Melitz 2003; Melitz and Ottaviano, 2008).[14]

3.5 Pareto Productivity Distribution

To obtain more analytical results, we now assume Pareto productivity distribution. Since c is the inverse of productivity, this implies that the c.d.f. of the cost distribution is given by $G(c) = (c/c_M)^\kappa$ for $c \in [0, c_M]$ and $\kappa \geq 1$. For the sake of conciseness, we further use $r = \hat{p}_H/\hat{p}_L$ to refer to the choke price ratio. The equilibrium prices rewrite as

$$p^*(c) = \begin{cases} (\alpha_H r + \alpha_L)^{1/2} \hat{p}_L^{1/2} c^{1/2} & \text{if} \quad c \leq \hat{c} \\ r^{1/2} \hat{p}_L^{1/2} c^{1/2} & \text{if} \quad c > \hat{c} \end{cases}, \tag{15}$$

while the cutoff cost as

$$\hat{c}^{1/2} = \frac{(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1 - \alpha_H^{1/2}} \hat{p}_L^{1/2}. \tag{16}$$

This gives the following three equilibrium conditions

$$0 = \Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right), \tag{17}$$

$$\hat{p}_L = c_M^{\frac{\kappa}{\kappa+1}} \left(\frac{f}{N}\right)^{\frac{1}{\kappa+1}} \left[\Gamma_2(r; \kappa, \alpha_H)\right]^{-\frac{1}{\kappa+1}}, \tag{18}$$

$$M = \frac{N s_L}{f} \frac{\Gamma_2(r; \kappa, \alpha_H)}{\Gamma_1(r; \kappa, \alpha_H)}. \tag{19}$$

where Φ, Γ_1 and Γ_2 are functions given in Appendix E. From (17) the value of the choke price ratio r only depends on the exogenous parameters κ, α_H, and the income ratio s_H/s_L. Given the value of r, one can determine the choke price $\hat{p}_L$ and the mass of entrants M from (18) and (19). The Pareto cost distribution permits to separate the effects of some parameters and sufficient statistics such as s_H/s_L and f/N. In terms of the effect of

[14]For example, in Melitz (2003), the homothetic preference implies that all that matters for selection is the mean (or total) income. In Melitz and Ottaviano (2008), the quasi-linear preference also implies the income elasticity of demand for differentiated goods is zero. That is, richer individuals spend the same amount on the differentiated products as the poor individuals, and they only spend more in the numeraire good.

income distribution, we show in Appendix E that r^* strictly increases in s_H/s_L. In Appendix E, we show that using this fact and Lemma 1, Γ_2 and Γ_2/Γ_1 are both strictly increasing in r^*. From these, it is shown that (13) holds. We have the following proposition.

Proposition 5. *Suppose a Pareto productivity distribution. Then,*

1. *There exists a unique equilibrium.*
2. *Assumptions (A0) and (A1) hold so that Proposition 4 holds.*
3. *The equilibrium choke price ratio r strictly increases with income inequality (higher s_H/s_L).*
4. *The number of entrants M is proportional to the population size N and inversely proportional to entry cost f.*
5. *The choke prices $(\hat{p}_L, \hat{p}_H)$, the cut-off cost $\hat{c}$, and the equilibrium price $p^*(c)$ of any firm with cost c increase in the maximum cost c_M and the entry cost f, and decrease with the population size N.*
6. *A proportional increase in incomes (i.e. higher s_L holding s_H/s_L unchanged) raises the mass of entrants M proportional to the average income.*

Proof. We show in Appendix E that (A0) and (A1) are satisfied by $G(c) = (c/c_M)^\kappa$ for $c \in [0, c_M]$ and $\kappa \geq 1$. Thus, by Lemma 1 and Propositions 3 and 4, Points 1 and 2 hold. For Point 3, also see Appendix E. Points 4-6 can be obtained by observing (17) and (19). ■

Under Pareto productivity distribution, the equilibrium utility can be written as

$$U(s_H) = \frac{M\hat{p}_L^\kappa}{c_M^\kappa}\left[a^\kappa \ln\left(r^{1/2}(\alpha_H r + \alpha_L)^{-1/2}\right) + \frac{r^\kappa}{2\kappa}\right] \tag{20}$$

$$U(s_L) = \frac{M\hat{p}_L^\kappa}{c_M^\kappa}\left[\frac{a^\kappa}{2\kappa} - a^\kappa \ln\left[\left(1-\alpha_H^{1/2}\right)a + (\alpha_H a r)^{1/2}\right]\right] \tag{21}$$

where $a \equiv \hat{c}/\hat{p}_L = \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right]^2 / \left(1-\alpha_H^{1/2}\right)^2$ with $1/r < a < 1 < r$. Using the equilibrium conditions, we get

$$\frac{M\hat{p}_L^\kappa}{c_M^\kappa} = s_L\left(\frac{N}{f}\right)^{\frac{1}{\kappa+1}} c_M^{-\frac{\kappa}{\kappa+1}} \frac{\left[\Gamma_2(r;\kappa,\alpha_H)\right]^{\frac{1}{\kappa+1}}}{\Gamma_1(r;\kappa,\alpha_H)}$$

So, utility rises with the larger population mass N and lower fixed input f. Those parameters indeed increase labor supply and decrease labor demand so that more firms enter and generate more product diversity and more competition, which benefit consumers.

Larger average productivity also raises utility as it can be easily shown that a decrease in c_M increases both the intensive and extensive margins. The utility differential between high and low income consumers is equal to

$$
\begin{aligned}
U(s_H) - U(s_L) &= \frac{M\hat{p}_L^\kappa}{2c_M^\kappa}\left\{a^\kappa \ln\left[r(\alpha_H r + \alpha_L)^{-1}\left[\left(1-\alpha_H^{1/2}\right)a + (\alpha_H a r)^{1/2}\right]^2\right] + \frac{r^\kappa - a^\kappa}{\kappa}\right\} \\
&= \frac{M\hat{p}_L^\kappa}{2c_M^\kappa}\left\{a^\kappa \ln\left[\frac{ar\left[\left(1-\alpha_H^{1/2}\right)a^{1/2} + (\alpha_H r)^{1/2}\right]^2}{\alpha_H r + \alpha_L}\right] + \frac{r^\kappa - a^\kappa}{\kappa}\right\}.
\end{aligned}
$$

As $ar > 1$, it can be easily shown that the logarithm term is positive. Hence, $U(s_H) > U(s_L)$ as $r > a$.

The effect income inequality on utility is not apparent from the above analytic, and we will further explore this in our quantitative analysis in Section 5.

4 Open Economy

We now study the implications of international trade and extend the above model to many trading countries and trade costs. We focus on the properties of income distribution and trade integration in the case of symmetric countries.

We consider n countries each with the same population size N and workers' skill distribution s_H and s_L with the probability α_H and $\alpha_L \in (0,1)$ $(\alpha_H + \alpha_L = 1)$. Earnings in each country are respectively ws_H and ws_L where w is the local wage. In each country, each firm produces a unique variety ω under monopolistic competition using an idiosyncratic marginal cost c, which yields a variable cost cw. Firms now produce for the local and foreign locations and incur an iceberg trade cost $\tau - 1 > 0$ per unit of exported good. That is, every unit of exported good costs τcw. Firms incur no trade cost on their local sales. They pay a cost wf to enter so that, in each country, a mass M of entrants obtains a distinct variety ω and draw a cost parameter c from the cumulative probability distribution G. Again, the measure of goods produced in each country $\mathrm{d}\omega$ is equal to $M\mathrm{d}G(c)$. One can then replace the label of a variety ω by its production cost c. Given the symmetric setting, all economic variables are equal and we can normalize all local wages to one.

Because of the symmetry, the aggregate demand for imports or local goods is given by the expression $Q(p)$ in (4). A firm producing with cost c makes a profit $(p-c)Q(p)$ for its home sales and $(p-\tau c)Q(p)$ for its exports. Under monopolistic competition, the firm chooses the prices that maximize its total profit taking all equilibrium choke prices as givens. At fixed choke prices, demands in each country are independent of each other so that optimal prices in a country are obtained independently of the prices in other countries (as in the closed economy model, see Proposition 1). Optimal local prices $p(c)$ write as before as

$$p^*(c) = \begin{cases} (\hat{p}_{HL}c)^{1/2} & \text{if} \quad c \leq \hat{c} \\ (\hat{p}_H c)^{1/2} & \text{if} \quad c > \hat{c} \end{cases},$$

where $\hat{c}$ is given by (6), while optimal export prices are simply given by $p^*(\tau c)$. The only difference is that the highest cost firm that sells to a foreign high (resp. low) income group has a cost equal to $\hat{p}_H/\tau$ (resp. $\hat{c}/\tau$). The equilibrium price levels and indices can be computed as before and reduced to the equilibrium conditions

$$M = \frac{s_H}{e_H(\hat{p}_H, \hat{p}_L)} = \frac{s_L}{e_L(\hat{p}_H, \hat{p}_L)}, \tag{22}$$

where

$$\begin{aligned} e_L(\hat{p}_H, \hat{p}_L) &= \int_0^{\hat{c}} \left(\hat{p}_L - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G(c) + (n-1) \int_0^{\hat{c}/\tau} \left(\hat{p}_L - \hat{p}_{HL}^{1/2} (\tau c)^{1/2}\right) \mathrm{d}G(c), \\ e_H(\hat{p}_H, \hat{p}_L) &= \int_0^{\hat{c}} \left(\hat{p}_H - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c) \\ &\quad + (n-1) \left[\int_0^{\hat{c}/\tau} \left(\hat{p}_H - \hat{p}_{HL}^{1/2} (\tau c)^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}/\tau}^{\hat{p}_H/\tau} \left(\hat{p}_H - \hat{p}_H^{1/2} (\tau c)^{1/2}\right) \mathrm{d}G(c)\right] \end{aligned}$$

express the consumers' average expenditure per available variety.

The firm's profit includes its home and foreign sales: $\pi(c) = (p^*(c) - c) Q(p^*(c)) + (n-1)(p^*(\tau c) - \tau c) Q(p^*(\tau c))$. The free entry implies that $\mathrm{E}[\pi(c)] = f$. We write this as

$$\pi(\hat{p}_H, \hat{p}_L) = \frac{f}{N}, \tag{23}$$

where $\pi(\hat{p}_H, \hat{p}_L) = \mathrm{E}[\pi(c)]/N$, or equivalently,

$$\begin{aligned} \pi(\hat{p}_H, \hat{p}_L) &= \int_0^{\hat{c}} \left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2 \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \alpha_H \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2 \mathrm{d}G(c) \\ &\quad + (n-1) \left[\int_0^{\hat{c}/\tau} \left(\hat{p}_{HL}^{1/2} - (\tau c)^{1/2}\right)^2 \mathrm{d}G(c) + \int_{\hat{c}/\tau}^{\hat{p}_H/\tau} \alpha_H \left(\hat{p}_H^{1/2} - (\tau c)^{1/2}\right)^2 \mathrm{d}G(c)\right]. \end{aligned}$$

The mass of surviving firms in a country is equal to $MG(\hat{p}_H)$.

As in the closed economy, the three market conditions in (22) and (23) determine the choke prices $(\hat{p}_H, \hat{p}_L)$ and mass of entrants M. The existence of an equilibrium can be proven in the same way. Therefore, we can conclude that *there always exists a trade equilibrium under condition (A1).*

Using Pareto productivity distribution and a similar procedure for simplifying equilibrium conditions, we obtain

$$0 = \Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right), \tag{24}$$

$$\hat{p}_L = c_M^{\frac{\kappa}{\kappa+1}} \left(\frac{f}{N\left[1+(n-1)\,\tau^{-\kappa}\right]}\right)^{\frac{1}{\kappa+1}} \left[\Gamma_2\left(r;\kappa,\alpha_H\right)\right]^{-\frac{1}{\kappa+1}}, \tag{25}$$

$$M = \frac{s_L N}{f}\frac{\Gamma_2\left(r;\kappa,\alpha_H\right)}{\Gamma_1\left(r;\kappa,\alpha_H\right)}, \tag{26}$$

where we used the definition $\hat{p}_H = r\hat{p}_L$ and Φ, Γ_1, and Γ_2 are the same functions as in the closed-economy model. Therefore, *Propositions 2, 4 and 5 also hold in the open economy.* In the following we investigate the equilibrium properties under Pareto productivity distribution.

4.1 Trade integration

What is the impact of trade integration? From equilibrium conditions (24) and (26), we observe that the trade cost τ and number of countries n do not affect the determination of the choke price ratio r and the mass of entrants M. Fixing all parameters except trade cost and number of countries, we can use conditions (24), (25) and (26) to write

$$\frac{\hat{p}_L}{\hat{p}_L^A} = \frac{\hat{p}_H}{\hat{p}_H^A} = \frac{\hat{p}_{HL}}{\hat{p}_{HL}^A} = \frac{\hat{c}}{\hat{c}^A} = \left(\frac{1}{1+(n-1)\,\tau^{-\kappa}}\right)^{\frac{1}{\kappa+1}} \quad \text{and} \quad \frac{M}{M^A} = 1, \tag{27}$$

where we denote the autarky situation with the superscript ^{A} (i.e. when $n=1$ or $\tau=\infty$).

The trade cost and number of countries only affect the choke price and cut-off through the term $(n-1)\,\tau^{-\kappa}$. Hence, an increase in the number of countries is equivalent to a decrease in trade cost such that

$$\Delta \ln \tau \; = -\frac{1}{\kappa}\Delta \ln\left(n-1\right)$$

where Δ denotes the difference in the variables between two cases. Accordingly, the adhesion of a new country in a trade network will be equivalent to a drop in trade costs. But, because of the term $n-1$ is not proportional, the equivalent drop in trade cost becomes larger in smaller trade networks. Since, larger trade networks and lower trade costs have the same effect on the choke prices $(\hat{p}_H, \hat{p}_L, \hat{p}_{HL})$ and cut-off $\hat{c}$, we can combine the discussion on trade costs and networks for all the variables that are directly determined by the term $(n-1)\,\tau^{-\kappa}$.

It can be seen from (27) that lowering trade costs diminishes both the choke prices $\hat{p}_L$ and $\hat{p}_H$ compared to the autarky situation. The prices of domestic sales relative to autarkic ones depend on whether the production cost of the good lies above or below the thresholds $\hat{c}$ and $\hat{c}^A$. More specifically, the ratio $p^*(c)/p^A(c)$ simplifies to $\left(\hat{p}_{HL}/\hat{p}^A_{HL}\right)^{1/2}$ if $c \leq \hat{c}$, $\left(\hat{p}_H/\hat{p}^A_{HL}\right)^{1/2}$ if $\hat{c} < c \leq \hat{c}^A$ and $\left(\hat{p}_H/\hat{p}^A_H\right)^{1/2}$ if $c > \hat{c}^A$. After some algebraic substitutions, we get

$$\frac{p^*(c)}{p^A(c)} = \begin{cases} \left(\frac{1}{1+(n-1)\tau^{-\kappa}}\right)^{\frac{1}{2(\kappa+1)}} & \text{if} \quad c \notin (\hat{c}, \hat{c}^A) \\ \left(\frac{r^A}{\alpha_H r^A + \alpha_L}\right)^{1/2} \left(\frac{1}{1+(n-1)\tau^{-\kappa}}\right)^{\frac{1}{2(\kappa+1)}} & \text{if} \quad c \in (\hat{c}, \hat{c}^A) \end{cases}.$$

So, a fall in trade cost reduces the prices of all domestic products except those with costs in the neighborhood of $\hat{c}$ for which prices jump upwards. The intuition is that a lower trade cost brings not only better market access and market sizes but also more competition. Competition reduces the choke prices and therefore the demands faced by every firm. Given lower demands, some firms are enticed to switch to the higher income group target. Similarly, the import prices relative to the autarky prices of firms with same cost c are given by

$$\frac{p^*(\tau c)}{p^A(c)} = \begin{cases} \tau^{1/2} \left(\frac{1}{1+(n-1)\tau^{-\kappa}}\right)^{\frac{1}{2(\kappa+1)}} & \text{if} \quad c \notin (\hat{c}/\tau, \hat{c}^A/\tau) \\ \tau^{1/2} \left(\frac{r^A}{\alpha_H r^A + \alpha_L}\right)^{1/2} \left(\frac{1}{1+(n-1)\tau^{-\kappa}}\right)^{\frac{1}{2(\kappa+1)}} & \text{if} \quad c \in (\hat{c}/\tau, \hat{c}^A/\tau) \end{cases}. \tag{28}$$

Thus, lower trade costs also reduce destination prices of imported varieties except for those goods produced at costs close to $\hat{c}/\tau$. The intuition is the same: lower trade costs increase competition so that product demands shift down and entice exporters to set lower prices unless they switch to the higher income group target. To sum up, *lower trade costs diminish destination prices of local and imported varieties except for those goods produced at costs close to $\hat{c}$ and $\hat{c}/\tau$.*

From the previous paragraph, we infer that imported goods are more expensive than local ones. Indeed, on the one hand, an importer producing abroad at a cost $c \notin (\hat{c}/\tau, \hat{c})$ sets its (destination) prices $\tau^{1/2}$ times higher than a local producer producing at the same cost. That is, $p^*(\tau c) = \tau^{1/2} p^*(c)$ if $c \notin (\hat{c}/\tau, \hat{c})$. On the other hand, a producer with cost $c \in [\hat{c}/\tau, \hat{c}]$ have different targets in their local and export markets: they target the whole population in local market but focus on the high income group in the export markets.

The difference between the prices of the local and imported goods is even higher. That is, $p^*(\tau c) = \tau^{1/2} p^*(c) > p^*(c)$ if $c \notin (\hat{c}/\tau, \hat{c})$.

From (27), a fall in trade cost does not affect entry M. However, the firm selection becomes tougher as the total mass of surviving firms in a country, $MG(\hat{p}_H)$, falls since $\hat{p}_H$ decreases. By contrast, the total number of goods consumed by the poor and the rich increase with smaller trade costs. Indeed, the total numbers of consumed goods are given by

$$|\Omega_H| = MG(\hat{p}_H) + (n-1)\, MG(\hat{p}_H/\tau) \quad \text{and} \quad |\Omega_L| = MG(\hat{c}) + (n-1)\, MG(\hat{c}/\tau).$$

Under Pareto cost distribution, this simplifies to

$$\frac{|\Omega_H|}{|\Omega_H^A|} = \frac{|\Omega_L|}{|\Omega_L^A|} = \left[1 + (n-1)\,\tau^{-\kappa}\right]^{\frac{1}{\kappa+1}},$$

where the superscript ^{A} again refers to autarky. Thus, *as trade costs fall, both the rich and the poor consume a wider set of goods*. However, the measures of their baskets of goods keep the same proportionality between each other.

All the previous results are valid for both lower trade costs and larger trade networks. Things are a little bit different for the variables that are not solely determined by the term $(n-1)\,\tau^{-\kappa}$. This is the case of the quantity of consumed goods and the number of varieties.

Let us first look at the number of local and imported goods sold to the whole local population: $MG(\hat{c})$ and $MG(\hat{c}/\tau)$. This is also the number of local and imported goods consumed by the poor. From (27), we see that the cut-off $\hat{c}$ always falls with smaller τ and larger n. Hence, trade integration leads to a reduction of the set of local goods consumed by the poor. Also, it readily comes that the cutoff $\hat{c}/\tau$ drops with larger n. This is because an enlargement of the trade network entices consumers to buy import from new countries and substitute for the existing imports. However, it can be checked that the cutoff $\hat{c}/\tau$ rises with smaller τ. The poor therefore purchase a wider set of foreign goods because those goods are less costly to ship. Hence, *lower trade costs and larger trade networks have opposite effects on the poor's basket of imports from a specific country.* However, it can be shown that the total set of imports $(n-1)\, MG(\hat{c}/\tau)$ expands with n.

We now look at the individual consumption. By (1), the individual consumption for a good by a rich or poor worker can be written as

$$q_h(c) = \left(\frac{\hat{p}_h}{p^*(c)} - 1\right), \quad h \in \{H, L\}$$

if $p^*(c) \leq \hat{p}_h$ and zero otherwise. The same expression with $p^*(\tau c)$ holds for imports. Since local goods are cheaper than imports, its is clear that individuals consumes larger quantities of the former than the latter. On the one hand, compared to autarky, the consumption of local goods produced at cost c is given by the following relationship:

$$\frac{q_h(c)+1}{q_h^A(c)+1} = \frac{\hat{p}_h}{\hat{p}_h^A}\frac{p^A(c)}{p^*(c)}, \quad h \in \{H, L\}. \tag{29}$$

Applying the above results yields

$$\frac{q_h(c)+1}{q_h^A(c)+1} = \begin{cases} \left(\frac{1}{1+(n-1)\tau^{-\kappa}}\right)^{\frac{1}{2(\kappa+1)}} & \text{if} \quad c \notin (\hat{c}, \hat{c}^A) \\ \left(\frac{r^A}{\alpha_H r^A + \alpha_L}\right)^{-1/2} \left(\frac{1}{1+(n-1)\tau^{-\kappa}}\right)^{\frac{1}{2(\kappa+1)}} & \text{if} \quad c \in (\hat{c}, \hat{c}^A) \end{cases}, \tag{30}$$

$h \in \{H, L\}$. Hence, *local consumption falls with lower trade costs and larger trade network.*

On the other hand, the consumption of imports produced at cost c is given by the relationship (29) with $p^*(\tau c)$ replacing $p^*(c)$. Because imported goods are more expensive than local ones, consumers purchase a lower quantity of each good. Indeed, the expressions in (30) must then be divided by $\tau^{1/2}$ and the cut-off intervals replaced by $(\hat{c}/\tau, \hat{c}^A/\tau)$. As a result, one can check that the $[q_h(\tau c)+1] / \left[q_h^A(c)+1\right]$ decreases with larger n but increases with smaller trade costs for any cost slightly away from the cost threshold $\hat{c}/\tau$. The presence of imports from a new country entices consumers to substitute for the existing imports. Also, a fall in trade cost reduces import prices and entices consumers to purchase a larger quantity of each imported good. Hence, *trade integration does not have the same effect on individual consumption of imports whenever it stems from lower trade cost or larger trade network.*

We summarize the above results in the following proposition:

Proposition 6. *Lower trade costs and larger trade networks (i) reduce product prices everywhere except those of the firms switching to target the higher income group, (ii) diminish the mass of surviving firms whereas the mass of entrants is unchanged, (iii) expand the set of goods consumed by both the rich and the poor, and (iv) reduce the quantity and the set of individual consumption of local goods. Also, lower trade costs increase the quantity and the set of individual consumption of imports whereas larger trade networks reduce them.*

From the derivation in Appendix G, equilibrium utility levels can be written as

$$U(s_H) = \frac{M\left(1+(n-1)\tau^{-\kappa}\right)\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[a^{\kappa}\ln\left(r^{1/2}\left(\alpha_H r+\alpha_L\right)^{-1/2}\right)+\frac{r^{\kappa}}{2\kappa}\right] \tag{31}$$

$$U(s_L) = \frac{M\left(1+(n-1)\tau^{-\kappa}\right)\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa}-a^{\kappa}\ln\left[\left(1-\alpha_H^{1/2}\right)a+\left(\alpha_H a r\right)^{1/2}\right]\right]. \tag{32}$$

By using (25) and comparing (20-21) and (31-32), we have

$$U(s_h) = \left(1+(n-1)\tau^{-\kappa}\right)^{\frac{1}{\kappa+1}} U^A(s_h),$$

where $h = H, L$, and $U^A(s_h)$ denotes the utility level under autarky. Trade integration therefore raises equilibrium utility levels of high and low income groups in the same proportion. The utility difference between those groups therefore rises in that same proportion.

How does the above utility increase due to trade integration compared to an increase in income? Let us fix income ratio s_H/s_L so that r and a remain constant. A percentage decrease in trade cost yields the same change in utility resulting from a percentage increase in average income if it satisfies the following relationship:

$$\left[\frac{d\ln U(s_h)}{d\ln\tau}\right]_{s_L \text{ fixed}} d\ln\tau = -\left[\frac{d\ln U(s_h)}{d\ln s_L}\right]_{\tau \text{ fixed}} d\ln s_L.$$

We have

$$\left[\frac{d\ln U(s_h)}{d\ln\tau}\right]_{s_L \text{ fixed}} = \frac{d\ln\left(1+(n-1)\tau^{-\kappa}\right)}{d\ln\tau}+\frac{d\ln\hat{p}_L^{\kappa}}{d\ln\tau} = -\frac{\kappa}{\kappa+1}\frac{(n-1)\tau^{-\kappa}}{1+(n-1)\tau^{-\kappa}}.$$

From (26) and (31), we observe that $U(s_h)$ is proportional to M, which, in turn, is proportional to s_L. Hence, we get

$$\left[\frac{d\ln U(s_h)}{d\ln s_L}\right]_{\tau \text{ fixed}} = 1.$$

Using the above results, we obtain

$$\mu \equiv \frac{d\ln s_L}{d\ln\tau} = \frac{\left[\frac{d\ln U(s_h)}{d\ln\tau}\right]_{s_L \text{ fixed}}}{-\left[\frac{d\ln U(s_h)}{d\ln s_L}\right]_{\tau \text{ fixed}}} = \frac{\kappa}{\kappa+1}\frac{(n-1)\tau^{-\kappa}}{1+(n-1)\tau^{-\kappa}}.$$

This figure is to be interpreted as follows: a one percent decrease in trade cost τ is equivalent to μ percent increase in average income, keeping constant the income inequality. Because utility $U(s_h)$ is proportional to income, μ is also the negative of the elasticity of utility to trade cost. For example, with $n = 2$, $\tau = 1.7$ and $\kappa = 3.03$,[15] we have $\mu = 0.13$. That is, a 1% fall in trade cost is equivalent to a 0.13% percent rise in average income, would income ratio (s_H/s_L) be constant. The effect is nevertheless stronger for lower trade costs and larger trade networks.

4.2 Income Distribution

We know by the structure of (24) to (26) that Proposition 4 hold in the open economy. That is, a mean-preserving spread of the income distribution increases the choke price of each country's high income group, reduces the country's (unweighted) average productivity and reduces the set of goods consumed by the poor relative to that by the rich in each country. The question then becomes whether lower trade costs amplify or attenuate the effect of a mean preserving spread of the income distribution.

A mean preserving spread of the income distribution (higher v at constant $\overline{s}$) raises the highest income s_H and reduces the lowest income s_L. In the open economy, each country (unweighted) average productivity is related to the opposite of its average cost $\int_0^{\hat{p}_H} cM\mathrm{d}G(c) \,/ \int_0^{\hat{p}_H} M\mathrm{d}G(c)$, or equivalently, $\int_0^{\hat{p}_H} c\mathrm{d}G(c) \,/ \int_0^{\hat{p}_H} \mathrm{d}G(c)$, which increases with a higher choke price $\hat{p}_H$ but is independent of the number of entrants, M. Hence, we need only to study the effect of a higher $\hat{p}_H$, irrespective of the numbers of entrants and firms. Because $\hat{p}_H = r\hat{p}_L$, we must discuss the effects of the mean preserving spread on r and $\hat{p}_L$ in equations (24) and (25). Let us consider the changes in income inequality s_H/s_L from s_H^a/s_L^a to s_H^b/s_L^b. By (24), the choke price ratio shifts from r^a to r^b where the superscripts ^{a} and ^{b} refer to the respective income ratios. By (25), we also have

$$\frac{\hat{p}_L^a}{\hat{p}_L^b} = \left(\frac{\Gamma_2(r^a;\kappa,\alpha_H)}{\Gamma_2(r^b;\kappa,\alpha_H)}\right)^{-\frac{1}{\kappa+1}},$$

which is independent of trade cost τ. Multiplying all terms by r_a/r_b we can write

$$\frac{\hat{p}_H^a}{\hat{p}_H^b} = \frac{r^a\hat{p}_L^a}{r^b\hat{p}_L^b} = \frac{r^a}{r^b}\left(\frac{\Gamma_2(r^a;\kappa,\alpha_H)}{\Gamma_2(r^b;\kappa,\alpha_H)}\right)^{-\frac{1}{\kappa+1}},$$

where the first equality stems from the definition of $r = \hat{p}_H/\hat{p}_L$. Hence the ratios of choke prices are also independent of trade costs τ. Finally, since we know that choke prices are lower for smaller trade costs, we can infer that the difference between the choke prices $\hat{p}_H^a$ and $\hat{p}_H^b$ must also be smaller for smaller trade costs. As a result, the average cost

[15] These parameter values are the ones adopted in our quantitative analysis in Section 5.

$\int_0^{\hat{p}_H} c\mathrm{d}G\left(c\right) / \int_0^{\hat{p}_H} \mathrm{d}G\left(c\right)$ increases less with lower trade cost when income inequality is higher. Since the opposite holds for the average productivity, we can state the following:

Proposition 7. *A mean preserving spread of income distribution reduces less each country's (unweighted) average productivity when trade costs are smaller.*

Stated differently, income redistribution from the rich to the poor improves each country's average productivity less under deeper trade integration. Also, the effects of income inequality on average productivity are the strongest under autarky ($\tau \to \infty$). The intuition is that trade expands product diversity and induces tougher firm selection. The selection cost-cutoffs are smaller under trade so that the mass of surviving firms $MG(\hat{p}_H)$ is smaller but those firms are more cost effective. The number of unsold goods is larger within the global economy than n autarkic economies. Hence, when the rich gets a higher income, she can spread her consumption towards the cheapest unsold goods in all countries rather than having to concentrate on the domestic unsold goods. In the end, consumed goods are produced at a lower cost.

5 Quantitative Analysis

5.1 Calibration

To calibrate the model, we attribute values to the nine parameters $(\alpha_H, s_L, s_H, c_M, f, \tau, \kappa, n, N)$. We calibrate on a 2015 US-like baseline economy where top 10% income individuals earn 7.9 more than the bottom 90% so that we set $\alpha_H^o = 0.10$ and $s_H^o/s_L^o = 7.9$ where the symbol o denotes the baseline values.[16] Without loss of generality, we can normalize the population size and the cost to unity such that $N^o = 1$ and $c_M^o = 1$. We focus on two (blocks of) symmetric countries ($n^o = 2$) and set the iceberg trade cost to the value $\tau^o = 1.7$ as estimated in Novy (2013).[17] We identify the model on three additional empirical relationships about firms' markups, survival and employment rates. First, following empirical

[16]The income ratio is calculated from top 10 percent income share data in 2014 for the US from the World Inequality Database.

[17]For examples, Novy (2013) estimated that the trade costs τ in 2000 between the US and Germany and between the US and the UK are 1.70 and 1.63, respectively. Using the same approach, the same set of estimates in 2014 reported by the World Bank's *International Trade Costs* data set are 1.723 and 1.704.

studies,[18] we impose an average markup on local sales $mrkup^o$ of 115%. That is,

$$mrkup^o = \frac{\kappa}{\kappa - 1/2}\left[\frac{\left[(\alpha_H^o r + \alpha_L^o)^{1/2} - r^{1/2}\right]\left[(\alpha_H^o r + \alpha_L^o)^{1/2} - \alpha_H^{o1/2} r^{1/2}\right]^{2\kappa-1}}{\left(1 - \alpha_H^{o1/2}\right)^{2\kappa-1} r^{\kappa}} + 1\right],$$

where the right-hand side is the unweighted average markup ($\int_0^{\widehat{p}_H} \frac{p}{c} dG$). This identity gives a relationship between κ and r as does the identity (24). Solving simultaneously those two identities allow us to pin down the values of r^o and κ^o. In turn, we get the values $\Gamma_1^o \equiv \Gamma_1(r^o; \kappa^o, \alpha_H^o)$ and $\Gamma_2^o \equiv \Gamma_2(r^o; \kappa^o, \alpha_H^o)$. We finally make use the value of the firm's survival rate $surv^o = G(\hat{p}_H)$ of 90%[19] and average employment per firm $empl^o = N/(MG(\hat{p}_H))$ of 66 workers as reported in the 2015 US census data ($148 * 10^6$ workers in $2.22 * 10^6$ firms having more than 5 employees). Using (25) and (26), we compute

$$surv^o = \left[\frac{1 + \tau_o^{-\kappa_o}}{f} r_o^{-(\kappa_o+1)} \Gamma_2^o\right]^{-\frac{\kappa_o}{\kappa_o+1}}$$

$$empl^o * surv^o = \frac{f}{s_L}\frac{\Gamma_1^o}{\Gamma_2^o},$$

which allow us to pin down f^o and s_L^o. This calibration process permits to recover the baseline economy parameter values $r^o = 1.539$, $\kappa^o = 3.03$, $f^o = 0.00887$ and $s_L^o = 0.00036$. In turn this yields $a^o = 0.860$ ($= \widehat{c}/\widehat{p}_L$). As an external validity check, we compute a 83% share of domestic expenditure on domestic goods, which fits well the reality of the US economy.[20]

5.2 Effects of Income Inequality

Table 2 presents the values of economic variables when workers incomes (or skills) increasingly spread about their mean. The first row presents the value of the income ratio that rises from 1 (second column) to the baseline model 7.9 (sixth column) and then to $3/2$ of this value (eighth column). The second row displays the respective values of the Gini coefficients. The next three lines show the value of skill endowment and their mean for the sake of completeness.

[18]For example, using Taiwanese manufacturing data and the markup-estimation approach by De Loecker and Warzynski (2012), Edmond, Midrigan, and Xu (2015) find an unweighted average markup of 1.13.

[19]We take the average exit rate as 0.1. See, for example, Klepper and Thompson (2006).

[20]Using information on domestic absorption and imports in Penn World Table 9.0, one can easily calculate the domestic expenditure share, and this share for the US in 2014 is 0.828.

s_H/s_L	1.	2.14	3.57	5.41	7.9	11.42	16.8
Gini	0.00	0.09	0.18	0.28	0.37	0.46	0.55
Top income share	0.10	0.19	0.28	0.38	0.47	0.56	0.65
s_L (thousands)	0.60	0.54	0.48	0.42	0.36	0.30	0.23
s_H (thousands)	0.60	1.16	1.71	2.26	2.82	3.37	3.93
Mean ($\alpha_H s_H + \alpha_L s_L$; thousands)	0.60	0.60	0.60	0.60	0.60	0.60	0.60
Ave. productivity (weighted by cost)	2.73	2.72	2.71	2.67	2.61	2.53	2.43
U_L vs. U_L base	1.55	1.41	1.28	1.14	1.00	0.86	0.71
U_H vs. U_H base	0.51	0.76	0.90	0.98	1.00	0.98	0.91
Equivalent variation L (relative to s_L^o)	0.69	0.51	0.34	0.17	0.00	-0.16	-0.33
Equivalent variation H (relative to s_H^o)	-0.30	-0.15	-0.06	-0.01	0.00	-0.02	-0.06

Table 2: Quantitative impact of mean preserving spreads of income.

In Section 3 our empirical analysis suggests a negative correlation between average productivity and income dispersion. The theoretical analysis in Section 3.4 shows that the average productivity falls with mean-preserving spreads. Because further analysis on productivity is analytically difficult, we resort to the quantitative analysis here. The seventh row in Table 2 shows reports the quantitative values for average productivity weighted by cost, given by

$$\frac{\int_0^{\hat{p}_H} (1/c) c Q\left(p^*(c)\right) dG(c)}{\int_0^{\hat{p}_H} c Q\left(p^*(c)\right) dG(c)},$$

which simplifies the total output over total cost. The observation of this row confirms our previous analyses: average productivity falls with stronger income inequality. Ceteris paribus, rich consumers purchase larger quantity per good and add goods that are more costly to make into their consumption basket as their income rises. As richer consumers buy more quantity and larger number of goods, their effect on total consumption dominates so that firms on average produce more costly goods.

The eighth and ninth lines of Table 2 compare the achieved utility levels compared to the baseline levels. The poor' utility monotonically falls with a mean preserving spread of income distribution as they get lower incomes. Interestingly, the rich's utility first increases and then decreases with higher income dispersion. Too strong income inequality may thus turn out to be a disadvantage for the rich. The intuition balances the effects of their larger purchasing power and larger number of expensive products. First, when income inequality strengthens, the rich get larger incomes and raise their demands so that they are willing to consume more in quantity and number of products. Second, income inequality reduces the poor' incomes and demands. This entices some firms with cost lower than $\hat{c}$ shift their consumer target from all individuals to only the rich ones. As a consequence, those firms raise their prices, which negatively affects the rich's consumption. In other words, for such products, the rich can no longer "hide behind the poor" and benefit from the low prices targeted to poorer people. Firms' price discrimination hits further the rich. Price hikes can be large because richer individuals have lower demand elasticity. One can then observe from the eighth row of Table 2 that the rich gain from larger income discrepancies only for income ratio s_H/s_L lower than the baseline level 7.9. Above that level, they are hurt by the above price hikes.

Finally, the last two rows of Table 2 display the *relative equivalent variations* as the percentage of additional income needed in the baseline model[21] to match the utility level obtained in another inequality configuration. To allow comparison, those measures take the baseline equilibrium price system and its product space as givens. Although they are a partial equilibrium measures, relative equivalent variations are better suited to express the magnitude of the impact of welfare inequality on the poor and rich. Hence, going from the sixth to the fifth column means to move from the baseline income ratio of 7.9 to 5.41. This implies a fall of 9 points in the Gini coefficient, a rise of the poor's income from 0.36 to 0.42 and a fall in the rich's income from 2.82 to 2.26, which amounts respectively for about 16% and -20% of their baseline incomes. However, the relative equivalent variations are 17% and -1% respectively for the poor and rich. Hence, the negative impact on the rich is much lower than her actual income change. In the same vein, going from the sixth to the second column implies the most drastic move from the baseline model to full income redistribution (equal incomes). Poor's income moves up from 0.36 to 0.60, that is, for about 67% of her baseline income. The rich's income moves down from 2.82 to 0.60, which is a 78% income drop. However, in terms of equivalent variations, the rich lose only 30% while the poor gain 69% of their purchasing power. Finally, going from the sixth to the seventh or eighth column implies higher inequality compared to the baseline model. Yet, this move harms *both* the poor and rich in terms of utility level and equivalent variation. This is undesirable for either the social planner or each income group.

5.3 Effects of Trade Cost

Table 3 presents main economic indicators for alternative trade costs, keeping the other parameters of the baseline model. Going from the left to the right hand, the columns successively report the cases of free trade, $\tau = 1$, a 10% fall in trade cost on the baseline

[21] That is, given the set of consumed goods and prices at the baseline.

model, the trade cost of the baseline model, $\tau = 1.7$, and the trade cost at the autarky limit, $\tau \to \infty$. (Weighted) average productivity falls with higher trade costs as less productive firms survive in the markets. It can be shown that this property also holds when average productivity is weighted by sales. Lowering trade cost by 10% from the baseline model augments productivity by 1.5% ($= 2.652/2.612 - 1$). Going from autarky to free trade increases average productivity by 16% (=$2.497/2.965 - 1$). The result that trade liberalization induces higher average productivity is consistent with the firm-selection literature *a la* Melitz (2003). The expenditure share on domestic goods falls as the economy moves from autarky to free trade. Lowering trade cost by 10% reduces it by 6% ($= 0.784/0.833 - 1$) and substantially raises the share of imports by 29% ($= (1 - 0.784)/(1 - 0.833) - 1)$).

The fourth and fifth rows show the low and high income groups' utility levels relative to their baseline level. As noted in Section 4.1, utility levels are affected in same proportions by trade cost if the income of each group is held constant. Lowering trade cost by 10% raises utility by 1.5% while moving from autarky to free trade increases it by about 18% ($1.135/0.956 - 1$). The negative of the elasticity of utility to trade cost is thus 0.15 ($= 1.5/1.5$), which is quite close to the point elasticity $\mu = 0.13$ mentioned in Section 4.1.

The last two rows display the relative equivalent variations. Lowering trade cost by 10% is equivalent to an increase of 1.8% of the poor's income and an increase of 1.0% of the rich's. Moving from autarky $\tau = \infty$ to the baseline trade cost $\tau = 1.7$ is equivalent to increases of poor and rich's real incomes by 5.4% ($= 1/(1 - 0.052) - 1$) and 2.9% ($= 1/(1 - 0.028) - 1$), respectively. Trade liberalization therefore benefits more to the poor because the poor consume more heavily on traded goods. A similar point was made by Fajgelbaum and Khandelwal (2016).

Trade cost τ	1.0	1.7×90%	1.7	∞
Ave. productivity (weighted by cost)	2.965	2.652	2.612	2.497
Expenditure share on domestic goods	0.5	0.784	0.833	1.
U_L vs. U_L base	1.135	1.015	1.000	0.956
U_H vs. U_H base	1.135	1.015	1.000	0.956
Equivalent variation L (relative to s_L^o)	0.160	0.018	0.000	-0.052
Equivalent variation H (relative to s_H^o)	0.087	0.010	0.000	-0.028

Table 3 Quantitative impact of trade cost

6 Conclusion

In this paper, we propose a theory of how income inequality may affect aggregate productivity and welfare in a global economy via selection under a non-homothetic preference with pro-competitive effects. We find that there is a negative cross effect of one group's income on the other group's consumption. We also find that a mean-preserving spread of income reduces average productivity (both weighted and unweighted) through the softening of firms' selection and the shuffling of the mass of consumption from low-cost to high-cost goods.

In the quantitative analysis, it is shown that a too large mean-preserving spread of income may harm the rich. Moreover, when measuring welfare in real terms by equivalent variation, we find that a reallocation of nominal income increases the poor's real income more than the fall of the rich's real income. Taken together, regardless of whether efficiency is measured in aggregate productivity or welfare, we find the contrary to the equity-efficiency trade-off is true in our model.

Our result that the negative effect of income inequality on average productivity is mitigated by international trade is intriguing because most theoretical and empirical studies point to the negative effect of globalization on equity. Of course there is actually no conflict because the directions of the causal relationships are different. Not only our model is consistent with the general understanding that trade helps the poor in terms of consumption, but it also suggests another positive side of trade: with higher income, the rich can spread their extra consumption over goods from various countries, instead of having to concentrate their consumption domestically under autarky. Collectively, a more efficient part of each country's cost distribution is sampled in the presence of trade.

Appendix A: Consumers' demands

Individuals are endowed with utility function $U = \int_{\omega\in\Omega} \ln(1+q(\omega))\,\mathrm{d}\omega$ over the commodity space $\Omega \subset \mathbb{R}$. Note that, firm entry limits the mass of commodities that are offered. Let $\overline{\Omega}$ be the set of commodities that are actually offered and associated with a price $p(\omega)$, $\omega \in \overline{\Omega}$. Other commodities $\omega \in \Omega\backslash\overline{\Omega}$ are not offered and cannot be consumed so that $q(\omega) = 0$ for $\omega \in \Omega\backslash\overline{\Omega}$. An individual in the income group h chooses the consumption $q(\omega)$, $\omega \in \overline{\Omega}$ that maximizes her utility U subject to her budget constraint $\int_{\omega\in\overline{\Omega}} p(\omega)\,q(\omega)\,\mathrm{d}\omega = s_h$. The Lagrangian function of individual h with income s_h is therefore defined as

$$\mathcal{L}_h = \int_{\omega\in\overline{\Omega}} \ln(1+q(\omega))\,\mathrm{d}\omega + \lambda_h\left(s_h - \int_{\omega\in\overline{\Omega}} p(\omega)\,q(\omega)\,\mathrm{d}\omega\right)$$

$\Omega_h \subseteq \mathbb{R}$. This is a concave function so that the following first order condition yields the consumer's best consumption choice:

$$\begin{array}{lll} \frac{\partial\mathcal{L}}{\partial q(\omega)} = \frac{1}{q(\omega)+1} - \lambda_h p(\omega) = 0 & \text{if} & q(\omega) > 0 \\ \frac{\partial\mathcal{L}}{\partial q(\omega)} = \frac{1}{q(\omega)+1} - \lambda_h p(\omega) < 0 & \text{if} & q(\omega) = 0 \end{array}$$

The set of consumed varieties is given by $\Omega_h \equiv \{\omega : q(\omega) > 0\} = \{\omega : p(\omega) < 1/\lambda_h\}$. For $\omega \in \Omega_h$, the first-order condition entails

$$q_h(\omega) = \frac{1}{\lambda_h p(\omega)} - 1,$$

and thus

$$\lambda_h = \frac{\int_{\omega\in\Omega_h} \mathrm{d}\omega}{s_h + \int_{\omega\in\Omega_h} p(\omega)\,\mathrm{d}\omega}.$$

Plugging λ_h back into the demand function, we obtain individual demand function

$$q_h(\omega) = \frac{\hat{p}_h}{p(\omega)} - 1,$$

where

$$\hat{p}_h \equiv \frac{1}{\lambda_h} = \frac{s_h + P_h}{|\Omega_h|}$$

is the choke price of consumer with income s_h, $P_h \equiv \int_{\omega\in\Omega_h} p(\omega)\,\mathrm{d}\omega$ is the aggregate price index for the goods consumed by s and $|\Omega(s)| = \int_{\omega\in\Omega_h} \mathrm{d}\omega$ is the measure of the set of goods consumed by individual h. Combining the above results, we obtain (1) and (2).

Note that the choke price $\hat{p}_h$ is the highest price that h is willing to pay to purchase any nonnegative amount of a variety. When s_h increases, λ_h falls and $\hat{p}_h$ rises so that Ω_h expands. As a result, one gets $s_H \geq s_L \iff s_H \leq s_L \iff \hat{p}_H \geq \hat{p}_L$.

Finally given that $q(\omega) = 0, \omega \notin \Omega_h$, the consumer's utility can successively be rewritten as

$$U_h = \int_{\omega\in\Omega_h} \ln\left(1 + q\left(\omega\right)\right) \mathrm{d}\omega + \int_{\omega\in\Omega\backslash\Omega_h} \ln\left(1\right) \mathrm{d}\omega = \int_{\omega\in\Omega_h} \ln\left(1 + q\left(\omega\right)\right) \mathrm{d}\omega$$

The indirect utility is thus equal to

$$V_h = \int_{\omega\in\Omega_h} \ln\left(\frac{s_h + P_h}{|\Omega_h|}\frac{1}{p\left(\omega\right)}\right) \mathrm{d}\omega \tag{33}$$

Appendix B: Firms' choices

The problem for a firm with cost c is

$$\begin{aligned}\max_p \ \pi &= \left(p - c\right) Q\left(p\right) \\ &= \begin{cases} \left(p - c\right) \alpha_H N \left(\frac{\hat{p}_H}{p} - 1\right) & \text{if} \quad p \in [\hat{p}_L, \hat{p}_H) \\ \left(p - c\right) N \left(\frac{\hat{p}_{HL}}{p} - 1\right) & \text{if} \quad p \in [0, \hat{p}_L) \end{cases}.\end{aligned}$$

For $p \in [0, \hat{p}_L)$, the firm sells to both groups and choose the price $p^*(c) = c^{1/2}\left(\hat{p}_{HL}\right)^{1/2}$ and markup $\left(\hat{p}_{HL}/c\right)^{1/2}$. The price increases and the markup decreases with higher marginal costs c, showing a pro-competitive effect. The firm gets a profit equal to $\pi^*_{HL}(c) = N\left[\left(\hat{p}_{HL}\right)^{1/2} - c^{1/2}\right]^2$. For $p \in [\hat{p}_L, \hat{p}_H)$, a firm sells only to high income consumers and set a prices $p^*(c) = c^{1/2}\hat{p}_H^{1/2}$ and markup $\left(\hat{p}_H/c\right)^{1/2}$. Prices increase and markups decrease in c. The firm gets a profit equal to $\pi^*_H(c) = \alpha_H N\left[\left(\hat{p}_H\right)^{1/2} - c^{1/2}\right]^2$. The firm chooses to charge $p^*(c) = c^{1/2}\left(\hat{p}_{HL}\right)^{1/2}$ if and only if $\pi^*_{HL}(c) \geq \pi^*_H(c)$, which is equivalent to

$$c^{1/2} \leq \hat{c}^{1/2} \equiv \frac{\left(\hat{p}_{HL}\right)^{1/2} - \left(\alpha_H\hat{p}_H\right)^{1/2}}{1 - \alpha_H^{1/2}}. \tag{34}$$

This argument yields (6) and (5). Observe that $p_H > p_{HL}$ for any c. So, there is upward jump of the price schedule $p^*(c)$ at $\hat{c}$.

In the product market equilibrium, it must be that each income group purchases the goods that are targeted to them. In particular, the low income consumers should buy only the goods produced at cost in the range $[0, \hat{c}]$. This means that their choke price $\hat{p}_L$ should satisfy $p^*(\hat{c}-0) < \hat{p}_L < p^*(\hat{c}+0)$. We show that this condition holds. Indeed, since $p^*(\hat{c}-0) = \hat{c}^{1/2}(\hat{p}_{HL})^{1/2}$ and $p^*(\hat{c}+0) = \hat{c}^{1/2}\hat{p}_H^{1/2}$ the previous condition becomes $\hat{c}^{1/2}(\hat{p}_{HL})^{1/2} < \hat{p}_L < \hat{c}^{1/2}\hat{p}_H^{1/2}$. Plugging the value of $\hat{c}$ and defining $r = \hat{p}_H/\hat{p}_L$ with $r > 1$ since $\hat{p}_H > \hat{p}_L$, we get the following inequalities

$$(\alpha_H r + \alpha_L) - (\alpha_H r(\alpha_H r + \alpha_L))^{1/2} < 1 - \alpha_H^{1/2} < ((\alpha_H r + \alpha_L) r)^{1/2} - (\alpha_H)^{1/2} r$$

Because $\alpha_H + \alpha_L = 1$, we have that the left-hand side and right-hand side are equal to the middle term for $r = 1$. It can be shown that the left-hand side falls with higher r while the right-hand side rises with it. Hence the inequalities are always satisfied.

For all goods to be supplied by firm with cost c to poor individuals, it must also that $c < \hat{p}_L$. This is obtained if $\hat{c} < \hat{p}_L$. Plugging the value of $\hat{c}$ and using $r = \hat{p}_H/\hat{p}_L$ we get the condition:

$$((\alpha_H r + \alpha_L))^{1/2} - (\alpha_H r)^{1/2} < 1 - \alpha_H^{1/2}$$

where the left-hand side decreases with larger r and is equal to the right-hand side at $r = 1$. So the condition is always satisfied.

Appendix C: Existence

The equilibrium is represented by the vector of variables $(\hat{p}_H, \hat{p}_L, M)$ with $\hat{p}_H \geq \hat{p}_L \geq 0$ and $M > 0$ that satisfy the market conditions (8) and (9) and entry conditions (12):

$$e_H(\hat{p}_H, \hat{p}_L) - \frac{s_H}{M} = 0 \tag{35}$$

$$e_L(\hat{p}_H, \hat{p}_L) - \frac{s_L}{M} = 0 \tag{36}$$

$$\pi(\hat{p}_H, \hat{p}_L) - \frac{f}{N} = 0 \tag{37}$$

where

$$e_H(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$

$$e_L(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$

are the consumers' average expenditures per available variety and

$$\pi\left(\hat{p}_H,\hat{p}_L\right)=\int_0^{\hat{p}_H}\max\left\{\left(\left(\alpha_H\hat{p}_H+\alpha_L\hat{p}_L\right)^{1/2}-c^{1/2}\right)^2,\alpha_H\left(\hat{p}_H^{1/2}-c^{1/2}\right)^2\right\}\mathrm{d}G\left(c\right)$$

is the expected operational profit before entry. In those equations $\hat{c}$ is implicitly given by the solution of

$$\hat{c}^{1/2}=\frac{\left(\hat{p}_{HL}\right)^{1/2}-\left(\alpha_H\hat{p}_H\right)^{1/2}}{1-\alpha_H^{1/2}}$$

with $\partial\hat{c}/\partial\hat{p}_H<0<\partial\hat{c}/\partial\hat{p}_H$. It can readily be shown that $\pi_H>0,\pi_L>0$ and $e_{LL}>0>e_{LH}$ where $e_{hl}=\partial e_h/\partial\hat{p}_l$ and $\pi_l=\partial\pi/\partial\hat{p}_l$, $h,l\in\{H,L\}$.

Using (35) and (36), we can rewrite the equilibrium conditions as

$$H(\hat{p}_H,\hat{p}_L,M)\equiv M-\frac{\alpha_H s_H+\alpha_L s_L}{\alpha_H e_H+\alpha_L e_L}=0 \tag{38}$$

$$F\left(\hat{p}_H,\hat{p}_L\right)\equiv\frac{e_H}{s_H}-\frac{e_L}{s_L}=0 \tag{39}$$

$$\Pi\left(\hat{p}_H,\hat{p}_L\right)\equiv\pi\left(\hat{p}_H,\hat{p}_L\right)-\frac{f}{N}=0 \tag{40}$$

The equilibrium is then given by the vector $(\hat{p}_H,\hat{p}_L,M)$ that solves (38), (39) and (40). Note that the choke prices are solutions of (39) and (40) while the mass of entrants is the solution of (38) at equilibrium choke prices.

To show the existence of the equilibrium, note that, since the e_H, e_L and π are continuous functions of $(\hat{p}_H,\hat{p}_L,M)$, the expressions in conditions (40), (38) and (39) are also continuous on R^{+3}. It then suffices to prove that each expression has opposite sign on two points in the support of $(\hat{p}_H,\hat{p}_L,M)\in R^{+3}$ with $\hat{p}_H\geq\hat{p}_L\geq0$ and $M>0$.

First, suppose that $(\hat{p}_H,\hat{p}_L,M)=(y,0,M)$. Then, $\hat{c}=0$ so that $e_H(y,0)=\int_0^y\left(y-y^{1/2}c^{1/2}\right)$ $\mathrm{d}G\left(c\right)>0$ and $e_L(y,0)=0$. We compute

$$\Pi(y,0)=\alpha_H\int_0^y\left(y^{1/2}-c^{1/2}\right)^2\mathrm{d}G\left(c\right)-\frac{f}{N}$$

$$H(y,0,M)=M-\frac{\alpha_H s_H+\alpha_L s_L}{\alpha_H\int_0^y\left(y-y^{1/2}c^{1/2}\right)\mathrm{d}G\left(c\right)}$$

$$F(y,0)=\frac{1}{s_H}\int_0^y\left(y-y^{1/2}c^{1/2}\right)\mathrm{d}G\left(c\right)$$

If y is small enough, we have $\Pi(y,0)<0$, $H(y,0,M)<0$ and $F(y,0)>0$.

Second, we consider that $G(c)$ has a bounded support and finite mean. That is, $G:[0,c_M]\rightarrow[0,1]$ such that $\mathrm{E}(c)=\int_0^{c_M}c\mathrm{d}G\left(c\right)<\infty$. We define $\hat{p}_L=x$, $\hat{p}_H=rx$, $\hat{p}_{HL}=$

$(\alpha_H r + \alpha_L)\, x$ and $\hat{c} = ax$ where $1 \le r < \infty$ and $a^{1/2} \equiv \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right] / \left(1 - \alpha_H^{1/2}\right) \in (0,1]$. We further set x such that $c_M < ax < x < rx$. This implies that $\int_0^{rx} \mathrm{d}G = \int_0^{c_M} \mathrm{d}G = 1$, $\int_0^{ax} c^{1/2}\mathrm{d}G = \int_0^{c_M} c^{1/2}\mathrm{d}G\,(c) = \mathrm{E}(c^{1/2})$, and $\int_{ax}^{rx} c^{1/2}\mathrm{d}G\,(c) = \int_{c_M}^{c_M} c^{1/2}\mathrm{d}G\,(c) = 0$. So, when $(\hat{p}_H, \hat{p}_L) = (rx, x)$, we have

$$e_H\,(rx, x) = rx - (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2})$$
$$e_L\,(rx, x) = x - (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2})$$

while

$$\Pi\,(rx, x) = \left[(\alpha_H r + \alpha_L)\, x - 2\,(\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2}) + \mathrm{E}(c)\right] - \frac{f}{N}$$
$$H(rx, x, M) = M - \frac{\alpha_H s_H + \alpha_L s_L}{(\alpha_H r + \alpha_L)\, x - (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2})}$$
$$F\,(rx, x) = \left(\frac{r}{s_H} - \frac{1}{s_L}\right) x + \left(\frac{1}{s_L} - \frac{1}{s_H}\right) (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2})$$

For x sufficiently large, it comes $\Pi\,(rx, x) > 0$ and $H(rx, x, M) > 0$ while $F\,(rx, x) < 0$ if $r < s_H/s_L$.

We can then choose five scalars, x large enough, y small enough, $r < s_H/s_L$, $M' > 0$ and $M'' > 0$, such that the functions Π, H and F have opposite signs at the points $(\hat{p}_H, \hat{p}_L, M) = (rx, x, M')$ and $(y, 0, M'')$. This proves the existence of an equilibrium.

Appendix D: Income and Demand

In this appendix, we show how changes in income affect choke prices. Differentiating totally (40) and (39), we get

$$\begin{bmatrix} e_{HH} s_H^{-1} - e_{LH} s_L^{-1} & e_{HL} s_H^{-1} - e_{LL} s_L^{-1} \\ \pi_H & \pi_L \end{bmatrix} \cdot \begin{bmatrix} \mathrm{d}\hat{p}_H \\ \mathrm{d}\hat{p}_L \end{bmatrix} = \begin{bmatrix} -e_H \mathrm{d}s_H^{-1} + e_L \mathrm{d}s_L^{-1} \\ 0 \end{bmatrix}$$

where $e_{hl} \equiv \partial e_h / \partial \hat{p}_l$ and $\pi_l \equiv \partial \pi / \partial \hat{p}_l$, $h, l \in \{H, L\}$. In Appendix C, it has been shown that $\pi_H > 0, \pi_L > 0$ and $e_{LL} > 0 > e_{LH}$. Under the assumption $e_{HH} > 0 > e_{HL}$, the determinant of the matrix in the above LHS, $\Delta = \left(e_{HH} s_H^{-1} - e_{LH} s_L^{-1}\right) \pi_L - \left(e_{HL} s_H^{-1} - e_{LL} s_L^{-1}\right) \pi_H$ is strictly positive. We have

$$\begin{bmatrix} \mathrm{d}\hat{p}_H / \mathrm{d}s_H^{-1} \\ \mathrm{d}\hat{p}_L / \mathrm{d}s_H^{-1} \end{bmatrix} = \frac{e_H}{\Delta} \begin{bmatrix} -\pi_L \\ \pi_H \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathrm{d}\hat{p}_H / \mathrm{d}s_L^{-1} \\ \mathrm{d}\hat{p}_L / \mathrm{d}s_L^{-1} \end{bmatrix} = \frac{e_L}{\Delta} \begin{bmatrix} \pi_L \\ -\pi_H \end{bmatrix}$$

Noting that $e_h = s_h/(M)$ by (35) and (36) so that $\left(\mathrm{d}\hat{p}_h/\mathrm{d}s_h^{-1}\right) = e_h\hat{p}_h M\left(\mathrm{d}\ln\hat{p}_h/\mathrm{d}\ln s_h^{-1}\right)$, $h = H, L$, we can rewrite the above expression as

$$\begin{bmatrix} \mathrm{d}\ln\hat{p}_H/\mathrm{d}\ln s_H \\ \mathrm{d}\ln\hat{p}_L/\mathrm{d}\ln s_H \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \pi_L \\ -\pi_H \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathrm{d}\ln\hat{p}_H/\mathrm{d}\ln s_L \\ \mathrm{d}\ln\hat{p}_L/\mathrm{d}\ln s_L \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -\pi_L \\ \pi_H \end{bmatrix}$$

We then get

$$\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H} = -\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_L} = \frac{1}{\Delta}\frac{\pi_L}{\hat{p}_H} > 0 \quad \text{and} \quad \frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_L} = -\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_H} = \frac{1}{\Delta}\frac{\pi_H}{\hat{p}_L} > 0 \tag{41}$$

Appendix E: Pareto productivity distribution

Equilibrium Conditions

Assume Pareto productivity, which translate to cost distribution with the c.d.f given by $G(c) = \left(\frac{c}{c_M}\right)^{\kappa}$ for $c \in [0, c_M]$ and $\kappa \geq 1$. The equilibrium is the vector $(\hat{p}_H, \hat{p}_L, M)$ that solves (40), (38) and (39). With some algebraic manipulations, these conditions are translated to

$$0 = \Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right) \tag{42}$$

$$\hat{p}_L = c_M^{\frac{\kappa}{\kappa+1}}\left(\frac{N}{f}\Gamma_2(r; \kappa, \alpha_H)\right)^{-\frac{1}{\kappa+1}} \tag{43}$$

$$M = \frac{N s_L}{f}\frac{\Gamma_2(r; \kappa, \alpha_H)}{\Gamma_1(r; \kappa, \alpha_H)}, \tag{44}$$

where

$$\begin{aligned}\Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right) \equiv{}& \frac{r^{\kappa+1}}{2\kappa+1}\frac{\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}}{\left[(\alpha_H r+\alpha_L)^{1/2}-\alpha_H^{1/2}r^{1/2}\right]^{2\kappa}} - \frac{s_H}{s_L}\left(1-\alpha_H^{1/2}\right) \\ &+ \frac{2\kappa\left[r^{1/2}+\left(\frac{s_H}{s_L}-1\right)(\alpha_H r+\alpha_L)^{1/2}\right]\left[(\alpha_H r+\alpha_L)^{1/2}-\alpha_H^{1/2}r^{1/2}\right]}{2\kappa+1}, \\ \Gamma_1(r; \kappa, \alpha_H) \equiv{}& \left(\frac{(\alpha_H r+\alpha_L)^{1/2}-\alpha_H^{1/2}r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa} - \frac{2\kappa(\alpha_H r+\alpha_L)^{1/2}}{2\kappa+1}\left(\frac{(\alpha_H r+\alpha_L)^{1/2}-\alpha_H^{1/2}r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa+1},\end{aligned}$$

and

$$
\begin{aligned}
\Gamma_2\left(r;\kappa,\alpha_H\right) = & \frac{\alpha_H r^{\kappa+1}}{\left(\kappa+1\right)\left(2\kappa+1\right)} + \alpha_L \left(\frac{\left(\alpha_H r+\alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa} \\
& + \frac{4\kappa\left[\alpha_H r^{1/2} - \left(\alpha_H r+\alpha_L\right)^{1/2}\right]}{2\kappa+1}\left(\frac{\left(\alpha_H r+\alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa+1} \\
& + \frac{\alpha_L\kappa}{\kappa+1}\left(\frac{\left(\alpha_H r+\alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa+2},
\end{aligned}
$$

Points 1 and 2 in Proposition 5

For Points 1 and 2, it suffices to show that (A0) and (A1) holds under Pareto productivity, as Lemma 1 and Propositions 3 and 4 can be therefore applied. Under $G\left(c\right) = \left(c/c_M\right)^{\kappa}$, where $c \in [0, c_M]$ and $\kappa > 1$, it is immediate that $\mathrm{E}(c) = \frac{\kappa}{c_M^{\kappa}}\int_0^{c_M} c^{\kappa}\mathrm{d}c = \frac{\kappa c_M}{\kappa+1} < \infty$, and hence (A0) holds. The next task is to show that $\partial e_H/\partial \hat{p}_H > 0$ and $\partial e_H/\partial \hat{p}_L < 0$. Observe that we can rewrite $e_H\left(\hat{p}_H, \hat{p}_L\right)$ as

$$
\begin{aligned}
e_H\left(\hat{p}_H,\hat{p}_L\right) &= \frac{\kappa}{c_M^{\kappa}}\left[\int_0^{\hat{c}}\left(\hat{p}_H - \left(\alpha_H\hat{p}_H+\alpha_L\hat{p}_L\right)^{1/2} c^{1/2}\right)c^{\kappa-1}\mathrm{d}c + \int_{\hat{c}}^{\hat{p}_H}\left(\hat{p}_H - \hat{p}_H^{1/2}c^{1/2}\right)c^{\kappa-1}\mathrm{d}c\right] \\
&\propto \hat{p}_H\int_0^{\hat{p}_H} c^{\kappa-1}\mathrm{d}c - \left[\left(\alpha_H\hat{p}_H+\alpha_L\hat{p}_L\right)^{1/2}\int_0^{\hat{c}} c^{\kappa-1/2}\mathrm{d}c + \hat{p}_H^{1/2}\int_{\hat{c}}^{\hat{p}_H} c^{\kappa-1/2}\mathrm{d}c\right] \\
&= \frac{1/2}{\kappa\left(\kappa+1/2\right)}\hat{p}_H^{\kappa+1} - \frac{1}{\kappa+1/2}\frac{\left(\hat{p}_{HL}^{1/2} - \hat{p}_H^{1/2}\right)\left[\left(\hat{p}_{HL}\right)^{1/2} - \left(\alpha_H\hat{p}_H\right)^{1/2}\right]^{2\kappa+1}}{\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}}.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&\partial e_H/\partial\hat{p}_L \\
&\propto -\frac{\alpha_L\hat{p}_{HL}^{-1/2}}{2}\left\{\left[\left(\hat{p}_{HL}\right)^{1/2} - \alpha_H^{1/2}\hat{p}_H^{1/2}\right]^{2\kappa+1} + \left(2\kappa+1\right)\left(\hat{p}_{HL}^{1/2} - \hat{p}_H^{1/2}\right)\left[\left(\hat{p}_{HL}\right)^{1/2} - \alpha_H^{1/2}\hat{p}_H^{1/2}\right]^{2\kappa}\right\} \\
&< 0.
\end{aligned}
$$

And,

$$
\begin{aligned}
&\partial e_H / \partial \hat{p}_H \\
&\propto \frac{1/2\,(\kappa+1)}{\kappa\,(\kappa+1/2)} \hat{p}_H^{\kappa} \\
&- \frac{\left(\frac{\alpha_H}{2}\hat{p}_{HL}^{-1/2} - \frac{1}{2}\hat{p}_H^{-1/2}\right) \times \left[\hat{p}_{HL}^{1/2} - (\alpha_H \hat{p}_H)^{1/2}\right] + \frac{\alpha_H(2\kappa+1)}{2}\left(\hat{p}_{HL}^{1/2} - \hat{p}_H^{1/2}\right)\left(\hat{p}_{HL}^{-1/2} - (\alpha_H\hat{p}_H)^{-1/2}\right)}{(\kappa+1/2)\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}\left[(\hat{p}_{HL})^{1/2} - (\alpha_H\hat{p}_H)^{1/2}\right]^{-2\kappa}}
\end{aligned}
$$

The above is positive if the second term is positive, that is, if

$$
\left(\frac{\alpha_H}{2}\hat{p}_{HL}^{-1/2} - \frac{1}{2}\hat{p}_H^{-1/2}\right) \times \left[\hat{p}_{HL}^{1/2} - (\alpha_H\hat{p}_H)^{1/2}\right] + \frac{\alpha_H\,(2\kappa+1)}{2}\left(\hat{p}_{HL}^{1/2} - \hat{p}_H^{1/2}\right)\left(\hat{p}_{HL}^{-1/2} - (\alpha_H\hat{p}_H)^{-1/2}\right) < 0.
$$

The above is true iff

$$
(2\kappa+2)\left(\alpha_H + \alpha_H^{1/2}\right) < \left[1 + (2\kappa+1)\,\alpha_H^{1/2}\right]\left(\frac{\alpha_H\hat{p}_H + \alpha_L\hat{p}_L}{\hat{p}_H}\right)^{1/2} + \left[\alpha_H^{3/2} + \alpha_H\,(2\kappa+1)\right]\left(\frac{\alpha_H\hat{p}_H + \alpha_L\hat{p}_L}{\hat{p}_H}\right)^{-}
$$

Let $y \equiv \left(\frac{\alpha_H\hat{p}_H + \alpha_L\hat{p}_L}{\hat{p}_H}\right)^{1/2} = \left(\alpha_H + \alpha_L\frac{\hat{p}_L}{\hat{p}_H}\right)^{1/2} \in (0,1)$. Thus, the above is true iff

$$
\left[1 + (2\kappa+1)\,\alpha_H^{1/2}\right] y^2 - (2\kappa+2)\left(\alpha_H + \alpha_H^{1/2}\right) y + \left[\alpha_H^{3/2} + \alpha_H\,(2\kappa+1)\right] > 0. \tag{45}
$$

As the determinant

$$
\begin{aligned}
\Delta &\equiv (2\kappa+2)\left(\alpha_H + \alpha_H^{1/2}\right)^2 - 4\left[1 + (2\kappa+1)\,\alpha_H^{1/2}\right]\left[\alpha_H^{3/2} + \alpha_H\,(2\kappa+1)\right] \\
&= -2\alpha_H\left[\left(2 + 6\kappa + 8\kappa^2\right)\sqrt{\alpha_H} + (1+3\kappa)\,\alpha_H + 3\kappa + 1\right] < 0,
\end{aligned}
$$

and $\left[1 + (2\kappa+1)\,\alpha_H^{1/2}\right] > 0$, (45) is true.

Comparative Statics of Income Distribution

The effect of s_H/s_L on r^*

Observe that

$$
\frac{\partial\Phi\,(r;\kappa,\alpha_H,x)}{\partial x} = -\left(1-\alpha_H^{1/2}\right) + \frac{2\kappa\,(\alpha_H r + \alpha_L)^{1/2}}{2\kappa+1}\left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right].
$$

The above is negative iff the following is negative

$$(\alpha_H r + \alpha_L) - (\alpha_H r + \alpha_L)^{1/2}\, \alpha_H^{1/2} r^{1/2} < \frac{(2\kappa+1)}{2\kappa}\left(1-\alpha_H^{1/2}\right). \tag{46}$$

Note that

$$\frac{d}{dr}\left[(\alpha_H r + \alpha_L) - (\alpha_H r + \alpha_L)^{1/2}\, \alpha_H^{1/2} r^{1/2}\right] = -\frac{2r\alpha_H^{\frac{3}{2}} + \sqrt{\alpha_H}\,(1-\alpha_H) - 2\sqrt{r}\alpha_H\sqrt{(1-\alpha_H)+r\alpha_H}}{2\sqrt{(1-\alpha_H)\,r + r^2\alpha_H}},$$

which is negative if and only if $1-\alpha_H > 0$, which is true. Hence, the upper bound of $(\alpha_H r + \alpha_L) - (\alpha_H r + \alpha_L)^{1/2}\, \alpha_H^{1/2} r^{1/2}$ is its value at $r=1$, $1-\alpha_H^{1/2}$. Hence, (46) is true, and equilibrium r^* strictly increases in $\frac{s_H}{s_L}$.

Γ_2 and Γ_2/Γ_1 are both strictly increasing in r^*

Next, we show that $\Gamma_2'(r^*) > 0$. Suppose $\Gamma_2'(r^*) \leq 0$, and consider an increase in s_H with s_L fixed. Then, r^* increases. By $\Gamma_2'(r^*) \leq 0$, equilibrium $\hat{p}_L$ increases or stays the same, and this in turn implies that equilibrium $\hat{p}_H$ increases. Thus, $\frac{d\hat{p}_H}{ds_H} > 0$. By the lemmas proved in Appendix D, $\frac{d\hat{p}_L}{ds_L} > 0$, $\frac{d\hat{p}_H}{ds_L} < 0$, and $\frac{d\hat{p}_L}{ds_H} < 0$. But $\frac{d\hat{p}_L}{ds_H} < 0$ implies that $\hat{p}_L$ decreases, which reaches a contradiction. The result follows.

Next, we show that $(\Gamma_2/\Gamma_1)'(r^*) > 0$. Suppose $(\Gamma_2/\Gamma_1)'(r^*) \leq 0$, and again consider an increase in s_H with s_L fixed. Then, r^* increases. By $\Gamma_2'(r^*) > 0$, equilibrium $\hat{p}_L$ decreases. Again, by the lemmas in Appendix D, $\hat{p}_H$ increases. As $e_{LL} > 0$ and $e_{LH} < 0$, e_L decreases. As $(\Gamma_2/\Gamma_1)'(r^*) \leq 0$, equilibrium M decreases or stays the same. Equilibrium condition $s_L/(M) = e_L$ is thus violated.

Appendix F: Production in International Trade

Firms differ in their marginal cost $w_i c$. Given equilibrium $\hat{p}_L$ and $\hat{p}_H$, the problem for a firm located in i with c is

$$\max_{\{p_{ij}\}_{j=1}^n \geq c} \pi_i(c) = \sum_j \left[p_{ij} - \tau_{ij} w_i c\right] Q_{ij}(p_{ij}; c).$$

This is equivalent to solving, in each market j,

$$\max_{p_{ij}\geq c} \pi_{ij}(c) = \begin{cases} \left[p_{ij} - \tau_{ij} w_i c\right] \alpha_H N_j \left(\frac{\hat{p}_{H,j}}{p_{ij}} - 1\right) & \text{if} \quad p_{ij} \in [\hat{p}_{L,j}, \hat{p}_{H,j}) \\ \left[p_{ij} - \tau_{ij} w_i c\right] N_j \left(\frac{\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j}}{p_{ij}} - 1\right) & \text{if} \quad p_{ij} \in [0, \hat{p}_{L,j}) \end{cases}.$$

For $p_{ij} \in [0, \hat{p}_{L,j})$,

$$\pi_{HL,ij}(c) = \max_{p_{ij}} [p_{ij} - \tau_{ij} w_i c] N_j \left(\frac{\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j}}{p_{ij}} - 1 \right),$$

which entails

$$\begin{aligned}
p_{ij,HL}(c) &= \tau_{ij}^{1/2} w_i^{1/2} c^{1/2} \left(\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j} \right)^{1/2}. \\
m_{ij,HL}(c) &\equiv \frac{p_{ijHL}(c)}{\tau_{ij} w_i c} = \left(\frac{\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j}}{\tau_{ij} w_i c} \right)^{1/2}. \\
\pi_{ij,HL}(c) &= N_j \left[\left(\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j} \right)^{1/2} - \left(\tau_{ij} w_i c \right)^{1/2} \right]^2.
\end{aligned}$$

For $p_{ij} \in [\hat{p}_{L,j}, \hat{p}_{H,j})$, the firms' problem is

$$\max_{p_{ij}} \pi_{H,ij}(c) = [p_{ij} - \tau_{ij} w_i c] \alpha_H N_j \left(\frac{\hat{p}_{H,j}}{p_{ij}} - 1 \right),$$

and the first-order condition entails

$$\begin{aligned}
p_{H,ij}(c) &= \left(\tau_{ij} w_i c \right)^{1/2} \hat{p}_{H,j}^{1/2}. \\
m_{H,ij}(c) &\equiv \frac{p_{H,ij}(c)}{\tau_{ij} w_i c} = \left(\frac{\hat{p}_{H,j}}{\tau_{ij} w_i c} \right)^{1/2}. \\
\pi_{H,ij}(c) &= \alpha_H N_j \left[\hat{p}_{H,j}^{1/2} - \left(\tau_{ij} w_i c \right)^{1/2} \right]^2.
\end{aligned}$$

The difference here from the closed economy model is that the existence of τ_{ij} raises prices but decreases markups, given $\hat{p}_{H,j}$ and $\hat{p}_{L,j}$.

Next, $\pi_{HL,ij}(c) - \pi_{H,ij}(c) > 0$ if and only if

$$c^{1/2} < \hat{c}_{ij}^{1/2} \equiv \frac{\left(\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j} \right)^{1/2} - \alpha_H^{1/2} \hat{p}_{H,j}^{1/2}}{\left(\tau_{ij} w_i \right)^{1/2} \left(1 - \alpha_H^{1/2} \right)}. \tag{47}$$

To sum up, the optimal price is

$$p_{ij}^*(c) = \begin{cases} p_{HL,ij}(c) = \tau_{ij}^{1/2} w_i^{1/2} c^{1/2} \left(\alpha_H \hat{p}_{H,j} + \alpha_L \hat{p}_{L,j} \right)^{1/2} & \text{if} \quad c \leq \hat{c}_{ij} \\ p_{H,ij}(c) = \left(\tau_{ij} w_i c \right)^{1/2} \hat{p}_{H,j}^{1/2} & \text{if} \quad c > \hat{c}_{ij} \end{cases}.$$

Note that $p_{H,ij}(c) > p_{HL,ij}(c)$ for any c. So there is upward jump of the price schedule p^* in terms of c at $\hat{c}_{ij}$.

Appendix G: Welfare in International Trade

Indirect utility is given by (33) or

$$U\left(s_h\right)=\int_{\omega\in\Omega_h}\ln\left(\frac{\hat{p}_h}{p^*\left(\omega\right)}\right)\mathrm{d}\omega.$$

The low income worker has a set of consumed goods Ω_L that includes the ranges $[0,M]\times[0,\hat{c}]$ and $[0,M]\times[0,\hat{c}/\tau]$ for local and imported goods. Using equilibrium prices p^*, $\hat{p}_L/p^*\left(\omega\right)=\hat{p}_L/(\hat{p}_{HL}c)^{1/2}$ and $\hat{p}_L/(\hat{p}_{HL}\tau c)^{1/2}$ for local and imported consumption, we get

$$U\left(s_L\right)=\int_0^{\hat{c}}\ln\left(\frac{\hat{p}_L}{(\hat{p}_{HL}c)^{1/2}}\right)M\mathrm{d}G(c)+(n-1)\int_0^{\hat{c}/\tau}\ln\left(\frac{\hat{p}_L}{(\hat{p}_{HL}\tau c)^{1/2}}\right)M\mathrm{d}G(c)$$

One can compute $\int\ln\left(Ac^{-1/2}\right)dG(c)=\frac{1}{2}\left(\frac{c}{c_M}\right)^{\kappa}\left[2\ln\left(A\right)+\frac{1}{\kappa}-\ln\left(c\right)\right]$ where A is a positive constant. Applying this to the above expression and simplifying we get$\left[1+\left(n-1\right)\tau^{-\kappa}\right]M\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa}-\right.$

$$U\left(s_L\right)=M\left[1+(n-1)\tau^{-\kappa}\right]\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa}-a^{\kappa}\ln\left[\left(1-\alpha_H^{1/2}\right)a+\left(\alpha_H ar\right)^{1/2}\right]\right]$$

where $r=\hat{p}_H/\hat{p}_L$ and $a=\hat{c}/\hat{p}_L$.

The high income worker has a set of consumed goods Ω_H that includes the ranges $[0,M]\times[0,\hat{p}_H]$ and $[0,M]\times[0,\hat{p}_H/\tau]$ for local and imported goods. Using equilibrium prices,we get

$$\begin{aligned}U\left(s_H\right)=&\int_0^{\hat{c}}\ln\left(\frac{\hat{p}_H}{(\hat{p}_{HL}c)^{1/2}}\right)M\mathrm{d}G(c)+\int_{\hat{c}}^{\hat{p}_H}\ln\left(\frac{\hat{p}_H}{(\hat{p}_{H}c)^{1/2}}\right)M\mathrm{d}G(c)\\&+(n-1)\left[\int_0^{\hat{c}/\tau}\ln\left(\frac{\hat{p}_H}{(\hat{p}_{HL}\tau c)^{1/2}}\right)M\mathrm{d}G(c)+\int_{\hat{c}}^{\hat{p}_H/\tau}\ln\left(\frac{\hat{p}_H}{(\hat{p}_{H}\tau c)^{1/2}}\right)M\mathrm{d}G(c)\right]\end{aligned}$$

Using the same procedure as above, this simplifies to

$$U\left(s_H\right)=M\left[1+(n-1)\tau^{-\kappa}\right]\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[a^{\kappa}\ln\left(r^{1/2}\left(\alpha_H r+\alpha_L\right)^{-1/2}\right)+\frac{r^{\kappa}}{2\kappa}\right].$$

Appendix H: Equivalent Variation

We define the *relative equivalent variation* to be the relative increase in income, $(\Delta s_h/s_h)^{\mathrm{eq}}$, that a worker h must receive to raise her utility level from the equilibrium utility U_h^* to the target utility level U_h taking as given the equilibrium price system $p^*(\omega)$, $\omega \in \Omega^*$ and its product space Ω^*. By (33), we can write worker h's indirect utility as

$$V_h(s_h) = \int_{\omega \in \Omega_h^*} \ln\left(\frac{\widehat{p}_h^*(s_h)}{p^*(\omega)}\right) \mathrm{d}\omega$$

where $\widehat{p}_h^*(s_h) = (s_h + P_h^*) / |\Omega_h^*|$ is the workers h's choke price expressed as a function of income s_h, Ω_h^* is her equilibrium set of purchased goods and $P_h^* = \int_{\omega \in \Omega_h^*} p^*(\omega)\,\mathrm{d}\omega$ her price index. Then, a relative increase in income $\Delta s_h / s_h$ implies an income change from s_h to $s_h + \Delta s_h$, which yields a change in utility level such that

$$U_h^* - U_h = \int_{\omega \in \Omega_h^*} \ln\left(\frac{\widehat{p}_h^*(s_h)}{\widehat{p}_h^*(s_h + \Delta s_h)}\right) \mathrm{d}\omega = |\Omega_h^*| \;\ln\left(\frac{s_h + P_h^*}{s_h + \Delta s_h + P_h^*}\right)$$

Inverting this expression, we obtain

$$\frac{\Delta s_h}{s_h + P_h^*} = \exp\left(-\frac{U_h^* - U_h}{|\Omega_h^*|}\right) - 1$$

Using the definition of $\widehat{p}_h^*(s_h)$, the relative equivalent variation can be expressed as

$$\left(\frac{\Delta s_h}{s_h}\right)^{\mathrm{eq}} = \frac{|\Omega_h^*|\, \widehat{p}_h^*(s_h)}{s_h}\left[\exp\left(\frac{U_h - U_h^*}{|\Omega_h^*|}\right) - 1\right] \tag{48}$$

The low income consumers have $\widehat{p}_L^*(s_L) = \widehat{p}_L$ while, under Pareto productivity distributions, the mass of varieties is given by $|\Omega_L^*| = MG(\widehat{c})\,[1 + (n-1)\tau^{-\kappa}]$. Then, their relative equivalent variation is given by the following formula:

$$\left(\frac{\Delta s_L}{s_L}\right)^{\mathrm{eq}} = \frac{MG(\widehat{c})\widehat{p}_L}{s_L}\left[1 + (n-1)\tau^{-\kappa}\right]\left[\exp\left(\frac{U_L - U_L^*}{MG(\widehat{c})\,[1 + (n-1)\tau^{-\kappa}]}\right) - 1\right]$$

The high income consumers with $\widehat{p}_H^*(s_H) = \widehat{p}_H$ and $|\Omega_H^*| = MG(\widehat{p}_H)\,[1 + (n-1)\tau^{-\kappa}]$ get

$$\left(\frac{\Delta s_H}{s_H}\right)^{\mathrm{eq}} = \frac{MG(\widehat{p}_H)\widehat{p}_H}{s_H}\left[1 + (n-1)\tau^{-\kappa}\right]\left[\exp\left(\frac{U_H - U_H^*}{MG(\widehat{p}_H)\,[1 + (n-1)\tau^{-\kappa}]}\right) - 1\right]$$

Note that for small utility differences, we can approximate relative equivalent variations as

$$\left(\frac{\Delta s_h}{s_h}\right)^{\mathrm{eq}} \simeq \frac{\widehat{p}_h}{s_h}\left(U_h - U_h^*\right), \quad h = L, H$$

Equivalent variations are proportional to utility differential and income group's choke price. Since $\widehat{p}_H > \widehat{p}_L$, a same rise in utility requires to give a larger increase in income to the higher income consumer because their marginal utility of consumption is lower. Finally, note that equivalent variations are partial equilibrium concepts because prices are fixed although they vary in our general equilibrium framework.

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